INVERSION SYMMETRY

Problem 1:

In Ch. 3 of the textbook, the invariance of Eq. (11) on page 35 under rotation is discussed. Suppose that we have a photonic crystal with inversion symmetry, implying that $\varepsilon(-\mathbf{r}) = \varepsilon(\mathbf{r})$ for the relative permittivity, and that the accompanying solutions of Maxwell's equations have odd or even parity: $\mathbf{H}(-\mathbf{r}, \omega) = \pm\mathbf{H}(\mathbf{r}, \omega)$.

a) Show that Maxwell's curl equations, Eq. (8) and (9) on page 10, imply that $\mathbf{E}(\mathbf{r}, \omega)$ and $\mathbf{H}(\mathbf{r}, \omega)$ must have opposite parity when the photonic crystal has inversion symmetry.

b) Show that Eq. (11) on page 35 is invariant under inversion of the Bloch vector $\mathbf{k}$ when $\varepsilon(\mathbf{r})$ has inversion symmetry.

c) What is the relationship between $\mathbf{H}_{-\mathbf{k}}(\mathbf{r}, \omega)$ and $\mathbf{H}_{\mathbf{k}}(\mathbf{r}, \omega)$ when $\varepsilon(\mathbf{r})$ has inversion symmetry?

Updated 12 september 2013.