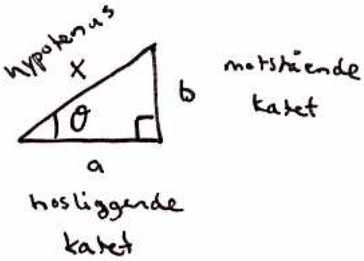
Trigonometri

Rettvinklet trekant



$$\boxed{\sin\theta = \frac{b}{x}\ , \quad \cos\theta = \frac{a}{x} \quad \text{og} \quad \tan\theta = \frac{b}{a} = \frac{\sin\theta}{\cos\theta}}$$

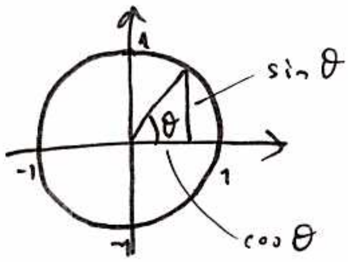
Pytagoras

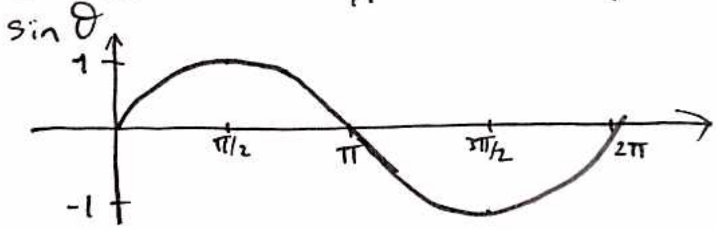
$$a^2 + b^2 = x^2$$

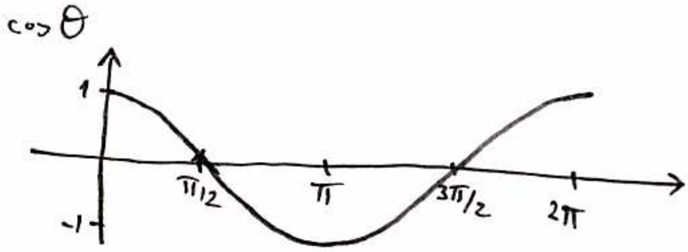
gir

$$\sin^2\theta + \cos^2\theta = 1$$

Enhetssirkelen (rettvinklet trekant med hypotenus lik 1)







Periodisitet

$$\sin(\theta + n\cdot 2\pi) = \sin\theta \qquad , n = \dots, -1, 0, 1, \dots$$

$$\cos(\theta + n\cdot 2\pi) = \cos\theta \qquad n = \dots, -1, 0, 1, \dots$$

Symmetri / Antisymmetri

$$\sin(-\theta) = -\sin\theta \qquad \text{Antisymmetrisk om } x = 0 \quad f(-x) = -f(x)$$

$$\cos(-\theta) = \cos\theta \qquad \text{Symmetrisk om } x = 0 \quad f(-x) = f(x)$$

Norm verdier

θ	$0°$ 0	$30°$ $\pi/6$	$45°$ $\pi/4$	$60°$ $\pi/3$	$90°$ $\pi/2$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1

Komplekse tall

Alle kompekse tall kan skrives på formen

$$z = a + ib$$

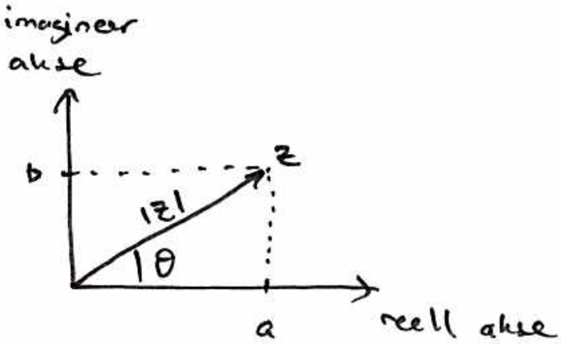
hvor $a, b \in \mathbb{R}$ og $i^2 = -1$. Kan også skrives på formen

$$z = |z| \cdot e^{i\theta}$$

hvor $|z| \in \mathbb{R}$ og hvor

$$e^{i\theta} = \cos\theta + i\sin\theta$$

(kan vises ved Taylor utvikling).



$$\sin\theta = \frac{b}{|z|}$$

$$\cos\theta = \frac{a}{|z|}$$

Kompleks konjugasjon

$$z^* \equiv a - ib$$

eller

$$z^* = |z|\, e^{-i\theta}$$

Dette gir

$$|z| = \sqrt{z \cdot z^*}$$

Eks $z_1 = 2 + 2i$ og $z_2 = 2 + i$

$$z_1 + z_2 = 4 + 3i$$

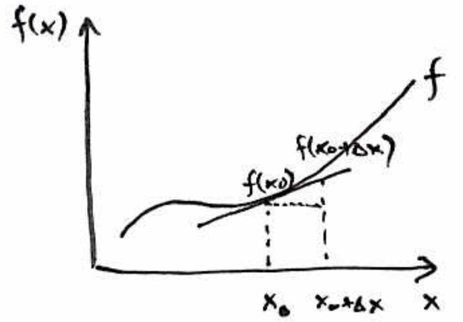
$$z_1 \cdot z_2 = (2+2i)(2+i) = 2 + 6i$$

$$z_1^* = 2 - 2i$$

$$z_1 \cdot z_1^* = 8 \quad , \quad \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \pi/4$$

$$z_1 = 2\sqrt{2} \cdot e^{i\pi/4}$$

Derivasjon



$f'(x)$ er stigningen til $f(x)$, dvs stigningen til tangenten:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

Derivasjon av elementære funksjoner

$$(x^n)' = n x^{n-1}$$

$$(a^x)' = \ln a \cdot a^x \qquad (a>0) \quad , \quad (e^x)' = e^x$$

$$(\ln |x|)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

Derivasjonsregler

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Kjerneregelen

$$\boxed{f(g(x))' = f'(g(x)) \cdot g'(x)}$$

Eks

$$\frac{d}{dx} \sin(x^2) = \cos(x^2) \cdot 2x$$

Oppgaver 23/8-2005

1. Gitt $z_1 = 2 - 2i$, $z_2 = \sqrt{3} - i$ og $z_3 = 1 + \sqrt{2}\, i$

a) Skriv ~~z_1 z_2 og z_3~~ z_1 og z_2 på formen $|z| \cdot e^{i\theta}$

b) Beregn $z_1 \cdot z_2$ og $z_2 + z_3$

2. Deriver følgende funksjoner

a) $\sin 2x$

b) $\tan(x^2)$

c) $\sin^2 x$

d) $e^{\sin x}$

e) $x \cdot e^{-x}$

f) $\sin x \cdot \cos x$

3. Finn de annenderiverte av

a) $x^2 \cdot e^{2x}$

b) $\sin x^2$

c) $\sin^3 x$

4. Avgjør hvorvidt følgende funksjoner er symmetrisk/antisymmetrisk om $x = 0$ eller ingen av delene

a) x^3

b) $x^2 \sin x$

c) $e^x \cos x$

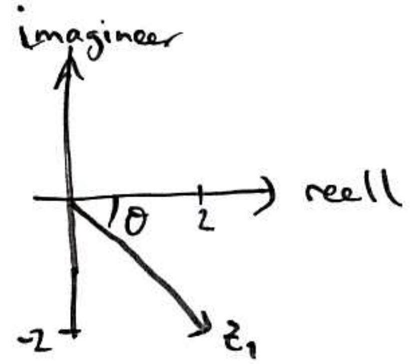
d) $\sin(x^2)$

Løsningsforslag oppgaver 23/8-2005

1. a) $z_1 = 2 - 2i$

$$z_1 \cdot z_1^* = (2-2i)(2+2i) = 2^2 - i^2 2^2 = 8$$

$$|z_1| = \sqrt{z_1 \cdot z_1^*} = \sqrt{8} = 2\sqrt{2}$$



$$\sin\theta = -\frac{2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{2}{2\sqrt{2}} = +\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \quad \theta = -\pi/4$$

Dette gir $z_1 = 2\sqrt{2} \cdot e^{-i\pi/4}$

$$z_2 = \sqrt{3} - i$$

$$|z_2| = \sqrt{3+1} = 2$$

$$\sin\theta = -\frac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2} \quad \Leftrightarrow \quad \theta = -\pi/6$$

Dette gir

$$z_2 = 2 \cdot e^{-i\pi/6}$$

b) $z_1 \cdot z_2 = 2\sqrt{2} \cdot e^{-i\pi/4} \cdot 2e^{-i\pi/6} = 4\sqrt{2} \cdot e^{-i(\pi/4 + \pi/6)}$

$$= 4\sqrt{2} \cdot e^{-i\frac{5}{12}\pi}$$

$$z_2 + z_3 = \sqrt{3} - i + 1 + \sqrt{2}\,i = \sqrt{3} + 1 + (\sqrt{2} - 1)\,i$$

② a) $(\sin 2x)' = \underline{\underline{2\cos 2x}}$

b) $\left(\tan(x^2)\right)' = \underline{\underline{\dfrac{1}{\cos^2(x^2)} \cdot 2x}}$

c) $(\sin^2 x)' = \underline{\underline{2\sin x \cdot \cos x}}$

d) $(e^{\sin x})' = \underline{\underline{\cos x \cdot e^{\sin x}}}$

e) $(x \cdot e^{-x})' = \underline{\underline{e^{-x} - x \cdot e^{-x}}}$

f) $(\sin x \cdot \cos x)' = \underline{\underline{\cos^2 x - \sin^2 x}}$

③ a) $(x^2 \cdot e^{2x})' = 2x \cdot e^{2x} + 2x^2 \cdot e^{2x}$

$(x^2 \cdot e^{2x})'' = (2x \cdot e^{2x} + 2x^2 \cdot e^{2x})'$

$= 2e^{2x} + 4xe^{2x} + 2(2x \cdot e^{2x} + 2x^2 \cdot e^{2x})$

$= \underline{\underline{2(x^2 + 4x + 1)\, e^{2x}}}$

b) $(\sin x^2)' = 2x \cos x^2$

$(\sin x^2)'' = 2\cos x^2 - (2x)^2 \sin x^2$

$= \underline{\underline{2\cos x^2 - 4x^2 \sin x^2}}$

c) $(\sin^3 x)' = 3\sin^2 x \cdot \cos x$

$(\sin^3 x)'' = 6\sin x \cdot \cos^2 x - 3\sin^3 x$

$= 6\sin x\,(1 - \sin^2 x) - 3\sin^3 x$

$= \underline{\underline{6\sin x - 9\sin^3 x}}$

4.

a) $f(x) = x^3$

$f(-x) = (-x)^3 = -x^3$ Antisymmetrisk

b) $f(x) = x^2 \sin x$

$f(-x) = (-x)^2 \sin(-x) = x^2(-\sin x) = -x^2 \sin x$

Antisymmetrisk

c) $f(x) = e^x \cos x$

$f(-x) = e^{-x} \cos(-x) = e^{-x} \cos x$

Ingen av delene

d) $f(x) = \sin(x^2)$

$f(-x) = \sin(x^2)$ Symmetrisk