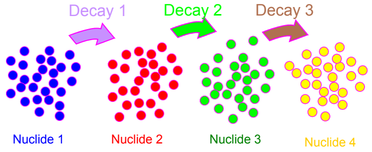
Genetically dependent nuclides

- When a radioactive nuclide disintegrates to a nucløide which in turn also is radioactive, we say that the two are genetically dependent
- There can be many consecutive nuclides in a genetic series, for instance: in the disintegration of ^{238}U, the nucleus ends in ^{206}Pb after 14 disintegrations.

Important example:

$$^{99}\text{Mo (66h)} \xrightarrow{\beta^-} {}^{99m}\text{Tc (6.0h)} \xrightarrow{IT} {}^{99}\text{Tc (213 000 y)}$$

Genetic dependence



For genetically dependent nuclides it is important to remember that **the same** atom changees all the time, and goes through different stages before ending up as stable.

Genetically dependent nuclides ctd.

Other important examples:

$$^{210}\text{Pb (22.3y)} \xrightarrow{\beta^-} {}^{210}\text{Bi (5.0d)} \xrightarrow{\beta^-}$$

$$^{210}\text{Po (138 d)} \xrightarrow{\alpha} {}^{206}\text{Pb (stab.)}$$

☹☺☹☺☹☺

$$^{49}\text{Cr (42m)} \xrightarrow{\beta+} {}^{49}\text{V (330d)} \xrightarrow{EC} {}^{49}\text{Ti (stable)}$$

♂♀♂♀♂

$$^{90}\text{Sr (29y)} \xrightarrow{\beta-} {}^{90}\text{Y (2.7d)} \xrightarrow{\beta-} {}^{90}\text{Zr (stable)}$$

カキクケサ

$$^{137}\text{Cs (30y)} \xrightarrow{\beta-} {}^{137m}\text{Ba (2.6m)} \xrightarrow{\gamma} {}^{137}\text{Ba (stable)}$$

Mother/daughter relations

- We have two genetically connected radionuclides, 1 and 2
- Nuclide 1 ⇒ nuclide 2 ⇒ stable

We want an expression of disintegration rates as function of time and start-conditions.

Assume that nuclide 1 is the first.
Then we have:
$N_1 = N_{1,0}e^{-\lambda_1 t}$
in the time interval dt, the increase in N2 is:
$dN_2 = (\lambda_1 N_1 - \lambda_2 N_2)dt$
or:

$$\frac{dN_2}{dt} + \lambda_2 N_2 - \lambda_1 N_{1,0}e^{-\lambda_1 t} = 0$$

Mother/daughter relations

Solve this differential equation:

$$N2 \equiv uv, \Rightarrow \frac{dN_2}{dt} = v\frac{du}{dt} + u\frac{dv}{dt}$$

$$v\frac{du}{dt} + u\frac{dv}{dt} + \lambda_2 uv - \lambda_1 N_{1,0}e^{-\lambda_1 t} = 0$$

Demand: $u\left(\frac{dv}{dt} + \lambda_2 v\right) = 0$

Gives: $v = e^{-\lambda_2 t}$

$$\frac{du}{dt}e^{-\lambda_2 t} - \lambda_1 N_{1,0}e^{-\lambda_1 t} = 0 \quad \text{or}$$

$$\frac{du}{dt} = \lambda_1 N_{1,0}e^{-(\lambda_1 - \lambda_2)t} \qquad \text{INTEGRATE}$$

$$u = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}e^{-(\lambda_1 - \lambda_2)t} + C$$

Mother/daughter relations.

$$N_2 = uv = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}e^{-\lambda_1 t} + Ce^{-\lambda_2 t}$$

C must be determined

$$N_{2,0} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0} + C$$

$$C = N_{2,0} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}$$

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}e^{-\lambda_2 t} + N_{2,0}e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_{2,0}e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1,0}e^{-\lambda_1 t}(1 - e^{-(\lambda_2 - \lambda_1)t}) + N_{2,0}e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(1 - e^{-(\lambda_2 - \lambda_1)t}) + N_{2,0}e^{-\lambda_2 t}$$

$$D_2 = \lambda_2 N_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} \underbrace{D_{1,0}e^{-\lambda_1 t}}_{D_1}(1 - e^{-(\lambda_2 - \lambda_1)t}) + D_{2,0}e^{-\lambda_2 t}$$

Mother/daughter relations.

Frequently, $N_{2,0}$ and $D_{2,0}$ are 0:

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(1-e^{-(\lambda_2-\lambda_1)t})$$

$$D_2 = \frac{\lambda_2}{\lambda_2 - \lambda_1} D_1(1-e^{-(\lambda_2-\lambda_1)t})$$

Saturation factor

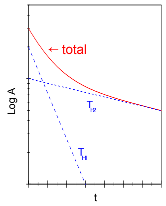
If $\lambda_1 << \lambda_2$:

$$N_2 = \frac{\lambda_1}{\lambda_2} N_1(1-e^{-\lambda_2 t})$$

$$D_2 = D_1(1-e^{-\lambda_2 t})$$

- Saturation factor:
  - 0,999 after 10 daughter nuclide halflives
  - Then $\lambda_1 N_1 = \lambda_2 N_2$ og $D_1 = D_2$
- With more steps in the chain and $T_{½}(1) >> T_{½}(2)$:
  - $\lambda_1 N_1 = \lambda_2 N_2 = \ldots\ldots\lambda_n N_{n1}$ and $D_1 = D_2 \ldots = D_n$

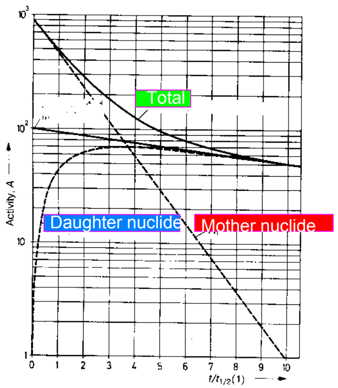
Genetic independence $T_{½}(1) << T_{½}(2)$



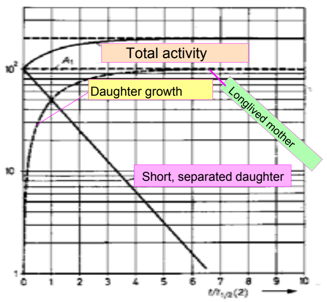
Mother/daughter relations, three cases

- Short mother, long daughter ($\lambda_1 >> \lambda_2$), $T_{½}(1) << T_{½}(2)$
  - No equilibrium
- Long mother, shorter daughter ($\lambda_1 < \lambda_2$), $T_{½}(1) > T_{½}(2)$
  - Transient equilibrium may occur
- Very long mother, short daughter ($\lambda_1 << \lambda_2$), $T_{½}(1) >> T_{½}(2)$
  - Secular equilibrium may occur

Genetically dependent nuclides $T_{½}(1) << T_{½}(2)$

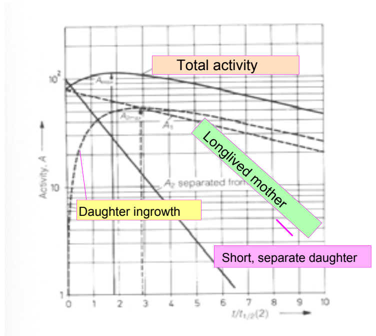


$T_{½}(1) >> T_{½}(2)$



- Equilibrium after approx.10 $T_{½}$
- The daughter nuclide may be chemically isolated, and reappears.

$T_{½}(1) > T_{½}(2)$, transient equilibrium



Also applicable as isotope generator.

F

Total

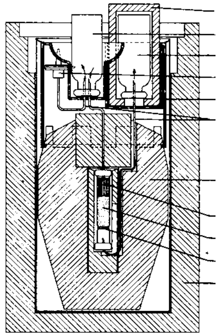
Datternuklide

Kor

"Isotope generator"

- An isotope generator is a system where a short-lived daughter nuclide (or a nuclide further sown in the sequence) is allowed to "grow in", whereafter it is separated from the mother activity utilising differences in chemical properties:
- Some useful examples
  - ^{99}Mo/^{99m}Tc
  - ^{68}Ge/^{68}Ga
  - ^{228}Th/..../^{212}Pb
  - ^{227}Ac/^{227}Th/^{223}Ra
  - ^{238}U/....../^{226}Ra/^{222}Rn
- The latter is a natural isotope generator used by Marie and Pierre Curie to obtain Ra from uranium-containing minerals.

^{99m}Tc-generator



Generator for elution of ^{99m}Tc (as water soluble $^{99m}TcO_4^-$) from unsoluble $^{99}Mo_2O_3$ (adsorbed on Al_2O_3)