Nuclear reactions
Production

Assume that we have a nuclear reaction leading to the production of a radionuclide.

Nuclide halflife: $T_{½}$.
Produktion rate: R (atoms s^{-1})

Consider the time element dt. At the actual time t we have N atoms of the type in question. The invrease in the number of atoms,dN, is then:

dN = Rdt - λNdt

Rdt atoms are formed, at the same time λNdt atoms disintegrate

Nuclear reactions
Production ctd.

We must solve the diff.eq:

$$\frac{dN}{d\tau} + \lambda N = R$$

Define: N= uv and obtain:

$$u\frac{dv}{d\tau} + v\left(\frac{du}{d\tau} + \lambda u\right) = R$$

=0 $\quad u = e^{-\lambda\tau}$

$$\frac{dv}{d\tau} = Re^{\lambda\tau} \qquad v = (\lambda^{-1}e^{\lambda\tau} + C)R$$

$$N = uv = (\lambda^{-1} + Ce^{-\lambda\tau})R$$

Boundary: N(0) = 0 : $\lambda^{-1} + C = 0$, $C = \lambda^{-1}$
$N = (R/\lambda)(1-e^{-\lambda\tau})$ og

$D = \lambda N = R\,(1-e^{-\lambda\tau})$

Nuclear reactions
Production ctd.

Note at saturation:(approx.10 $T_{½}$):
D = R

Frequently, one uses τ for irradiation time and **t** for decay time

Totally this gives the formula:

$D = \lambda N = R\,(1-e^{-\lambda\tau})e^{-\lambda t}$

Includes both irradiation time and decay time after the end of irradiation.

Nuclear reactions
Notation

Nuclear reactions generally have the form:

X + a ➜ Y + b

X - target
a - projectile
Y,b - products (may be many)

The notation normaly used is then:

X(a,b)Y

As Y, one normally chooses the most heavy product. This is normally easy, although there may be cases where there is doubt.
As a one **always** defines the projectile, even if it should happen to be heavier than the target nuclide.

Nuclear reactions
Energies

Q-verdien for en kjernereaksjon er:
Q = M(X) + M(a) - (M(Y) +M(b))

Q>0 - energy released (exotermic)
Q<0 - energy consumed (endothermic)

Q corresponds to -ΔH in chemical reactions. Note the difference in sign compared to “normal” chemistry

The Q-value is always given between the ground states of neutral atoms and particles, unless otherwise stated.

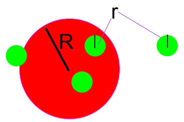
Nuclear reactions
Some projectiles

- p - protons
- n - neutrons
- d-deutrons
- ^{3}He-ions
- α-particles
- Heavy ions ($^{6}Li^{+}$ - $^{238}U^{92+}$)
- Electrons
- Mesons (π^{+}, π^{-})

Beware of the conservation laws in nuclear reactions!

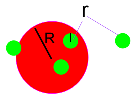
Nuclear reactions
Cross section

The cross section is an important concept dealing with nuclear reactions.



We imagine that we irradiate nuclei with radius R with projectiles having radius r. The projectiles come in normal to the plane. We consider firstly what happens if we imagine the nuclei and projectiles as classical spheres.

Nuclear reactions
Cross section

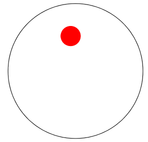


The process is defined by a surface, if the projectile hits the nucleus inside this surface, we get an event. If it passes outside of the surface, nothing happens. This surface is the cross section for the process, σ, amd is in thios case given as:

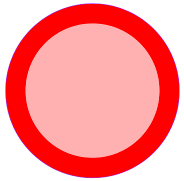
$$\sigma = \pi(r + R)^2$$

In nuclear chemistry, the probability of a particular reaction is given as such an imaginary "surface".

Nuclear reactions
Cross section

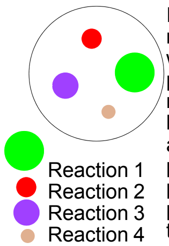


If a particular reaction occurs with low probability, its cross section will only constitute a small part of the total surface.



If a reaction is highly probable, its cross section can be larger than the extenstion of the nucleus, due to quantum mechanical effects

Nuclear reactions
Cross section



If several different reaction can happen with the same projectile, each reaction is represented by its "surface", and the area of the surface is proportional to the probability for that particular reaction to take place.

We also ofte operate with differential cross section $d\sigma/d\Omega$, as a function of the angle between the outgoing and ingoing particle. $d\Omega$ is the size of a differential space angle.