10.1

$f(x) = -\frac{2}{x}$

3 a) $y' \underbrace{- \frac{2}{x}}_{f(x)} y = \underbrace{x^2}_{g(x)}$ på $(0, \infty)$ $F(x) = -2\ln x$

$y(x) = e^{-F(x)}\left(\int e^{F(x)} g(x)\,dx + C\right)$

$= e^{2\ln x}\left(\int e^{-2\ln x} x^2\,dx + C\right)$

$= x^2\left(\int x^{-2} x^2\,dx + C\right)$

$= x^2\left(\int 1\,dx + C\right) = x^2(x + C) = \underline{\underline{x^3 + Cx^2}}$

d) $y' + \frac{2}{x} y = \frac{\arctan x}{x^2}$ på $(0, \infty)$

$f(x) = \frac{2}{x}$

$g(x) = \frac{\arctan x}{x^2}$

$F(x) = 2\ln x$

$y(x) = e^{-F(x)}\left(\int e^{F(x)} g(x)\,dx + C\right)$

$= e^{-2\ln x}\left(\int e^{2\ln x} \frac{\arctan x}{x^2}\,dx + C\right)$

$= x^{-2}\left(\int x^2 \frac{\arctan x}{x^2}\,dx + C\right)$

$= x^{-2}\left(\int \arctan x\,dx + C\right)$

$= x^{-2}\left(x \arctan x - \frac{1}{2}\ln(1+x^2) + C\right)$

$= \underline{\underline{\frac{\arctan x}{x} - \frac{1}{2x^2}\ln(1+x^2) + \frac{C}{x^2}}}$

$\int 1 \cdot \arctan x\,dx$

$= x \cdot \arctan x - \int x \frac{1}{1+x^2}\,dx$

$u = 1 + x^2 \Rightarrow du = 2x\,dx$

$= x \arctan x - \frac{1}{2}\int \frac{1}{u}\,du$

$= x \arctan x - \frac{1}{2}\ln|u|$

$= x \arctan x - \frac{1}{2}\ln(1+x^2)$

10.2.1 $y(t)$ befolkning i antall

$y'(t) = \underbrace{\text{tilskudd pga befolkningsvekst}} + \underbrace{\text{tilskudd pga innvandring}}$

$= \underbrace{0.02\,y(t)} + 40\,000$

$y'(t) \underbrace{-0.02}_{f(t)} y(t) = \underbrace{40\,000}_{g(t)}$ $F(t) = -0.02t$

$y(t) = e^{-F(t)}\left(\int e^{F(t)} g(t)\,dt + C\right)$

$= e^{0.02t}\left(\int e^{-0.02t} 40000\,dt + C\right)$ $\underbrace{e^{0.02t - 0.02t}} = e^0 = 1$

$= e^{0.02t}\left(e^{-0.02t}\frac{40000}{-0.02} + C\right) = e^{0.02t} e^{-0.02t}\frac{40000}{-0.02} + Ce^{0.02t}$

$= \underline{-2\,000\,000 + Ce^{0.02t}}$ $= \frac{40000}{-0.02} + Ce^{0.02t}$

$\frac{40\,000}{-\frac{1}{50}} = 40000 \cdot 50$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

$y(0) = 200\,0000$

$\Rightarrow -2\,000\,000 + C = 2000\,000$

$\Rightarrow C = 4\,000\,000$

$\Rightarrow \underline{\underline{y(t) = -2000\,000 + 4}}000\,000\,e^{0.02t}$

Oppg. 10.2, 5, 10, 15

10.2.5: $y(t)$: prosentandel av skadelig stoff i søppel på en midlertidig lagringsplass.

a) $y'(t) = -0.05\,y(t)$ ← 5 prosent av y i nedbrytning.

$y(0) = 2$

2 prosent farlig stoff ved $t = 0$

$$\frac{200\,000}{100\,00\,000} = \frac{1}{50} = 0.02 \to 2 \text{ prosent}$$

Ser at $y(t) = Ce^{-0.05t}$ er den generelle løsningen.

$y(0) = 2 \Rightarrow 2 = Ce^{0} \Rightarrow C = 2$

$$\Rightarrow \underline{\underline{y(t) = 2e^{-0.05t}}}$$

b) $z'(t) = \underbrace{-0.1z(t)}_{\text{nedbrytning på }10\%} + \underbrace{0.01e^{-0.05t}}_{\text{tilskudd}}$

$\underline{z(0) = 0}$ → har ikke rukket overføre noe enda ved $t = 0$

Når vi overfører en halv million tonn søppel ved tid t, så overfører vi $\frac{2e^{-0.05t}}{100} \cdot 0.5$ millioner tonn skadelig stoff.

$$\frac{e^{-0.05t}}{100} = \boxed{0.01\,e^{-0.05t}}$$

c) Kan skrives $z'(t) + 0.1z(t) = 0.01\,e^{-0.05t}$

$f(t) = 0.1$

$F(t) = 0.1t$

$g(t) = 0.01\,e^{-0.05t}$

formel:

$$z(t) = e^{-0.1t}\left(\int e^{0.1t}\,0.01e^{-0.05t}\,dt + C\right)$$

$$= e^{-0.1t}\left(\int 0.01e^{0.05t} + C\right) = e^{-0.1t}\left(\frac{0.01}{0.05}e^{0.05t} + C\right)$$

$$= e^{-0.1t}\left(0.2e^{0.05t} + C\right) = \underline{0.2e^{-0.05t} + Ce^{-0.1t}}$$

$z(0) = 0 \Rightarrow 0 = 0.2 + C \Rightarrow C = -0.2$

$$\Rightarrow z(t) = 0.2e^{-0.05t} - 0.2e^{-0.1t} = \underline{\underline{0.2\left(e^{-0.05t} - e^{-0.1t}\right)}}$$

$$\int e^{at}\,dt = \frac{1}{a}e^{at} + C$$

13.5.1 komp.

$$x'' + \sin(t x') - x^2 = e^t$$

sett $x_1 = x$, og $x_2 = x'$ $(\Rightarrow x_2' = x'')$

$$x_2' + \sin(t x_2) - x_1^2 = e^t \Rightarrow \boxed{\begin{aligned} x_1' &= x_2 \\ x_2' &= -\sin(t x_2) + x_1^2 + e^t \end{aligned}}$$

13.6.4: Eulers metode på et system av likninger.

$$f(t, \vec{x}) = \begin{pmatrix} x_2 \\ -\sin(t x_2) + x_1^2 + e^t \end{pmatrix}$$