Oppg 7 eks. 2016

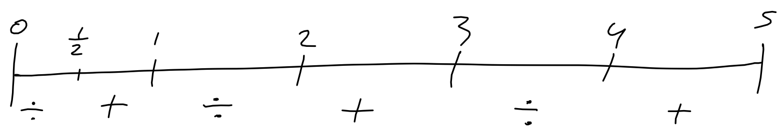
$$X_3 = x_2 - \frac{x_2 - x_1}{\underbrace{(x_2^2-2)}_{f(x_2)} - \underbrace{(x_1^2-2)}_{f(x_1)}} \overbrace{(x_2^2 - 2)}^{f(x_2)}$$

$$= 2 - \frac{2-1}{(2^2-2)-(1^2-2)}(2^2-2)$$

$$= 2 - \frac{1}{2-(-1)} \cdot 2 = 2 - \frac{2}{3} = \underline{\underline{\frac{4}{3}}}$$

Oppg 7 eks 2015

$$f(x) = (x-1)\left(x-\tfrac{1}{2}\right)(x-2)(x-3)(x-4)$$



første iterasjon: $[0, 5]$ $\quad f(0) < 0 \quad f(5) > 0$

midtpunktet $\quad f(2.5) > 0$

halveringsmetoden velger $[0, 2.5]$

andre iterasjon: midtpunkt: 1.25 $\quad f(0) < 0$

$f(1.25) < 0$

$f(2.5) > 0$

halveringsmetoden velger $[1.25, 2.5]$

Kun nullpunktet 2 ligger i $[1.25, 2.5]$, så halveringsmetoden finner denne.

10.4.7 i kompendiet: $\rightarrow x = \frac{1}{R}$ er eneste nullpunkt.

$f(x) = \frac{1}{x} - R \qquad f'(x) = -\frac{1}{x^2}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\frac{1}{x_n} - R}{-\frac{1}{x_n^2}} \frac{\cdot x_n^2}{\cdot x_n^2} = x_n + \left(\frac{1}{x_n} - R\right) x_n^2$$

$$= x_n + x_n - R x_n^2$$

$$= 2x_n - R x_n^2 = \underline{\underline{x_n(2 - R x_n)}}$$

Regne i pausen:

10.2	2ab	
10.3	2	komp.

10.5 i kompendiet

3c) $y'' - 4y' - y = 0 \qquad y(1) = 2 \qquad y'(1) = -1$

karakteristiske ligning: $r^2 - 4r - 1 = 0$

$$r = \frac{4 \pm \sqrt{4^2 - 4\cdot(-1)}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2} = \underline{2 \pm \sqrt{5}}$$

Generell løsning: $\underline{\underline{y(x) = Ce^{(2+\sqrt{5})x} + De^{(2-\sqrt{5})x}}}$

$y(1) = 2$ $\qquad \boxed{Ce^{2+\sqrt{5}} + De^{2-\sqrt{5}} = 2}$

$$y'(x) = C(2+\sqrt{5})e^{(2+\sqrt{5})x} + D(2-\sqrt{5})e^{(2-\sqrt{5})x}$$

$y'(1) = -1$ $\qquad \boxed{C(2+\sqrt{5})e^{(2+\sqrt{5})} + D(2-\sqrt{5})e^{2-\sqrt{5}} = -1}$

$$\underbrace{2Ce^{2+\sqrt{5}} + 2De^{2-\sqrt{5}}} + \sqrt{5}(Ce^{2+\sqrt{5}} - De^{2-\sqrt{5}}) = -1$$

$$2\cdot 2 + \sqrt{5}(Ce^{2+\sqrt{5}} - De^{2-\sqrt{5}}) = -1$$

$$\sqrt{5}(Ce^{2+\sqrt{5}} - De^{2-\sqrt{5}}) = -5$$

$$\boxed{Ce^{2+\sqrt{5}} - De^{2-\sqrt{5}} = -\sqrt{5}}$$

$$\left.\begin{aligned} Ce^{2+\sqrt{5}} + De^{2-\sqrt{5}} &= 2 \\ Ce^{2+\sqrt{5}} - De^{2-\sqrt{5}} &= -\sqrt{5} \end{aligned}\right\}$$

legg sammen: $2Ce^{2+\sqrt{5}} = 2 - \sqrt{5}$

$$\underline{C = \frac{1}{2}e^{-2-\sqrt{5}}(2-\sqrt{5})}$$

trekk fra: $2De^{2-\sqrt{5}} = 2 + \sqrt{5}$

$$\underline{D = \frac{1}{2}e^{-2+\sqrt{5}}(2+\sqrt{5})}$$

$$y(x) = Ce^{(2+\sqrt{5})x} + De^{(2-\sqrt{5})x}$$

$$= \frac{1}{2}(2-\sqrt{5})e^{-2-\sqrt{5}}e^{(2+\sqrt{5})x} + \frac{1}{2}(2+\sqrt{5})e^{-2+\sqrt{5}}e^{(2-\sqrt{5})x}$$

$$\underline{\underline{= \frac{1}{2}(2-\sqrt{5})e^{(2+\sqrt{5})(x-1)} + \frac{1}{2}(2+\sqrt{5})e^{(2-\sqrt{5})(x-1)}}}$$

10.5

3a) $y'' - 5y' + 4y = 0$ $\quad y(0) = 2 \;, \; y'(0) = -4$

$r^2 - 5r + 4 = 0$

$r = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \Rightarrow r_1 = 4 \;, r_2 = 1$

$y(x) = Ae^{4x} + Be^{x}$ $\quad y'(x) = 4Ae^{4x} + Be^{x}$

$y(0) = 2$

$y'(0) = -4$

$\left.\begin{aligned} A + B &= 2 \\ 4A + B &= -4 \end{aligned}\right\} \div$ $\quad 3A = -4 - 2 = -6$

$\underline{A = -2}$

$-2 + B = 2 \Rightarrow \underline{\underline{B = 4}}$

10.6.1

a) $y'' - y' - 2y = 0$

$r_1 = -1, \quad r_2 = 2$

$\underline{y(x) = Ce^{-x} + De^{2x}}$

b) $y'' - y' - 2y = e^x$

partikulær løsning $y_p = Ae^x$ $\qquad y_p' = y_p'' = Ae^x$

$Ae^x - Ae^x - 2Ae^x = e^x$

$-2Ae^x = e^x \Rightarrow A = -\frac{1}{2}$

$\underline{\underline{y_p = -\frac{1}{2}e^x}}$

c) $y'' - y' - 2y = e^x \qquad y(0) = y'(0) = 2$

$y = y_p + y_h = \underline{-\frac{1}{2}e^x + Ce^{-x} + De^{2x}}$ $\rightarrow$ derivert: $-\frac{1}{2}e^x - Ce^{-x} + 2De^{2x}$

$y(0) = 2: \quad -\frac{1}{2} + C + D = 2$

$y'(0) = 2: \quad -\frac{1}{2} - C + 2D = 2$

legg sammen: $-1 + 3D = 4$

$3D = 5$

$-\frac{1}{2} + C + \frac{5}{3} = 2 \Rightarrow \cdots \Rightarrow \underline{C = \frac{5}{6}} \qquad \underline{D = \frac{5}{3}}$

$\underline{\underline{y(x) = -\frac{1}{2}e^x + \frac{5}{6}e^{-x} + \frac{5}{3}e^{2x}}}$