11.2.6

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2}$$

c mellom 0 og x

$$\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$$

$$e^x = \underbrace{1+x+\frac{x^2}{2!}+\dots+\frac{x^n}{n!}}_{T_n e^x}+\frac{e^c}{(n+1)!}x^{n+1}$$

$$e^x-1-x = \frac{x^2}{2!}+\dots+\frac{x^n}{n!}+\frac{e^c}{(n+1)!}x^{n+1}$$

antar $n \geq 2$

$$\frac{e^x-1-x}{x^2} = \frac{1}{2!}+\dots\frac{x^{n-2}}{n!}+\frac{e^c}{(n+1)!}x^{n-1}$$

$$\boxed{\frac{e^x-1-x}{x^2} = \frac{1}{2!}+\frac{\overset{\text{begrenset}}{e^c}}{3!}\overset{0}{x}}$$

La $x\to 0$: HS går mot $\frac{1}{2!}=\frac{1}{2}$

11.2.15 $\quad g(x) = \sqrt[3]{1+x} = (1+x)^{\frac{1}{3}}$

a) Skrive opp Taylor av grad 2 om $x = 0$

b) Vis: for $x \geq 0$: $|\underbrace{R_2(x)}_{R_{g_2}(x)}| \leq \frac{5}{81}x^3$

c) Finn $\sqrt[3]{1003}$ med 7 desimalers nøyaktighet.

a): $g'(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}} \quad g''(x) = -\frac{2}{9}(1+x)^{-\frac{5}{3}} \quad g'''(x) = \frac{10}{27}(1+x)^{-\frac{8}{3}}$

$\underline{g(0) = 1} \quad \underline{g'(0) = \frac{1}{3}} \quad g''(0) = \underline{-\frac{2}{9}}$

$T_2(x) = g(0) + g'(0)x + \frac{g''(0)}{2!}x^2 = 1 + \frac{1}{3}x - \frac{1}{9}x^2$

b) $R_2(x) = \frac{g'''(c)}{3!}x^3 = \frac{10}{27 \cdot 3!}(1+c)^{-\frac{8}{3}}x^3$

$= \frac{5}{81}\underbrace{(1+c)^{-\frac{8}{3}}}x^3 \leq \frac{5}{81}x^3$

avtagende funksjon i c, og siden c er mellom 0 og x, så er dette $< (1+0)^{-\frac{8}{3}} = 1$

c) $\sqrt[3]{1003} = \sqrt[3]{1000+3} = \sqrt[3]{1000(1+3\cdot 10^{-3})} = 10\sqrt[3]{1+3\cdot 10^{-3}} = 10\,g(0.003)$

Feilen, $R_2(x)$, for $x = 0.003$ er, ifølge b), begrenset av

$|R_2(0.003)| \leq \frac{5}{81}(0.003)^3 = \frac{5}{81}(3\cdot 10^{-3})^3 = \frac{5}{81}27\cdot 10^{-9} = \frac{5}{3}10^{-9}$

Feilen for $10\,g(0.003)$ blir $\leq \frac{5}{3}10^{-8}$, som svarer til 7 riktige siffer.

Vår tilnærming blir $10 \cdot T_2(0.003) = 10\cdot\left(1 + \frac{1}{3}\cdot 3\times 10^{-3} - \frac{1}{9} \times 9\times 10^{-6}\right)$

$= 10(1 + 10^{-3} - 10^{-6}) = \underbrace{10 + 0.01} - 10^{-5}$

$= \underline{10.00999}$

9.2.3 i kompendiet

a) $P_3(x) = c_0(x-1)(x-3)(x-4) + c_1 x(x-3)(x-4) + c_2 x(x-1)(x-4) + c_3 x(x-1)(x-3)$

setter inn $x=0$: $c_0(0-1)(0-3)(0-4) = c_0(-1)(-3)(-4) = \underline{-12c_0 = 1}$

$x=1$: $c_1 \cdot 1 \cdot (1-3)(1-4) = c_1 \cdot (-2)(-3) = \underline{6c_1 = 0}$

$x=3$: $c_2 \cdot 3 \cdot (3-1)(3-4) = c_2 \cdot 3 \cdot 2 \cdot (-1) = \underline{-6c_2 = 2}$

$x=4$: $c_3 \cdot 4 \cdot (4-1)(4-3) = c_3 \cdot 4 \cdot 3 \cdot 1 = \underline{12c_3 = 1}$

Ser da at: $c_0 = -\frac{1}{12}$, $c_1 = 0$, $c_2 = -\frac{1}{3}$, $c_3 = \frac{1}{12}$

$\Rightarrow \underline{\underline{P_3(x) = -\frac{1}{12}(x-1)(x-3)(x-4) - \frac{1}{3}x(x-1)(x-4) + \frac{1}{12}x(x-1)(x-3)}}$

b) $P_3(x) = c_0 + c_1 x + c_2 x(x-1) + c_3 x(x-1)(x-3)$

$x=0$: $\boxed{c_0 = 1}$

$x=1$: $c_0 + c_1 = 0$

$x=3$: $c_0 + 3c_1 + c_2 3(3-1) = c_0 + 3c_1 + 6c_2 = 2$

$x=4$: $c_0 + 4c_1 + c_2 \cdot 4 \cdot (4-1) + c_3 \cdot 4(4-1)(4-3) = c_0 + 4c_1 + 12c_2 + 12c_3 = 1$

Ser da at: $c_0 = 1$

$c_0 + c_1 = 0 \Rightarrow c_1 = -c_0 = -1$

$c_0 + 3c_1 + 6c_2 = 2 \Rightarrow c_2 = \frac{2 - c_0 - 3c_1}{6} = \frac{2-1+3}{6} = \frac{4}{6} = \frac{2}{3}$

$c_0 + 4c_1 + 12c_2 + 12c_3 = 1 \Rightarrow c_3 = \frac{1 - c_0 - 4c_1 - 12c_2}{12}$

$= \frac{1 - 1 + 4 - 12 \cdot \frac{2}{3}}{12} = \frac{4-8}{12} = \frac{-4}{12} = -\frac{1}{3}$

$\Rightarrow P_3(x) = \underline{\underline{1 - x + \frac{2}{3}x(x-1) - \frac{1}{3}x(x-1)(x-3)}}$