9.1.2 c)

$f(x) = \sin x - \frac{2x}{c+x^2}$ $\qquad f(0) = 0$

Vil velge c slik at $T_3(x) = \frac{x^3}{3}$

$f'(x) = \cos x - \frac{2(c+x^2) - 2x \cdot 2x}{(c+x^2)^2} = \cos x - \frac{2c - 2x^2}{(c+x^2)^2}$

$T_3(x) = \underline{f(0)} + \underline{f'(0)}x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3$

$= \quad 0 \qquad 0 \qquad\qquad \frac{x^3}{3}$

$f'(0) = 1 - \frac{2c}{c^2} = 1 - \frac{2}{c} = 0 \Rightarrow c = 2$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

$-\frac{2x}{c+x^2} = -\frac{2x}{c}\frac{1}{1+\frac{x^2}{c}} = -\frac{2x}{c}\left(1 - \frac{x^2}{c} + \left(\frac{x^2}{c}\right)^2 - \dots\right)$

$\Rightarrow \sin x - \frac{2x}{c+x^2} = x - \frac{x^3}{3!} - \frac{2x}{c} + \frac{2x^3}{c^2} + \dots$

$= x - \frac{x^3}{3!} - \frac{2x}{2} + \frac{2x^3}{4} = \underline{x} - \frac{x^3}{6} - \underline{x} + \frac{x^3}{2} = \underline{\frac{x^3}{3}}$

11.2.3

optimal h^*

12.2.2 og 12.3.2 $f(x) = x^2$

Regn ut $\int_0^1 x^2 \, dx$ med 2 delintervaller med midtpunktsm. / trapesmetoden

(selve integralet blir $\frac{1}{3}$)

intervallene: $[0, \frac{1}{2}]$ og $[\frac{1}{2}, 1]$ $h = \frac{1}{2}$

midtpunkt: midtpunktene blir $\frac{1}{4}$ og $\frac{3}{4}$

$$\frac{1}{2}\left(f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)\right) = \frac{1}{2}\left(\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2\right) = \frac{1}{2}\left(\frac{1}{16} + \frac{9}{16}\right)$$

$$= \frac{1}{2} \cdot \frac{10}{16} = \underline{\underline{\frac{5}{16}}}$$

Trapes: $h\left(\frac{f(\overset{x_0}{a}) + f(\overset{x_n}{b})}{2} + \sum_{i=1}^{n-1} f(x_i)\right)$

$$= \frac{1}{2}\left(\frac{f(0) + f(1)}{2} + f\left(\frac{1}{2}\right)\right) = \frac{1}{2}\left(\frac{0^2 + 1^2}{2} + \left(\frac{1}{2}\right)^2\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{4}\right) = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$