10.6.2

a) $y'' - 2y' - 8y = 0$

$r^2 - 2r - 8 = 0 \Leftrightarrow r = \frac{2 \pm \sqrt{4 + 4 \cdot 8}}{2} = \frac{2 \pm 6}{2} = 1 \pm 3$

røttene er $r_1 = 4$, $r_2 = -2$

Generell løsning: $\underline{\underline{Ce^{4x} + De^{-2x}}}$

b) $y'' - 2y' - 8y = 6 - 8x$

Vi prøver $y_p(x) = Ax + B$: $y'' = 0$ $y' = A$

$-2A - 8(Ax + B) = \underline{-8Ax} - 2A - 8B = 6 - \underline{8x}$

$A = 1$

$-2A - 8B = 6$

$-2 - 8B = 6$

$-8 = 8B$

$B = -1$

$\Rightarrow y_p = Ax + B = x - 1$ er partikulær løsning.

c) generell løsning for $y'' - 2y' - 8y = 6 - 8x$:

$y(x) = Ce^{4x} + De^{-2x} + x - 1$

$y(1) = 0$, $y'(1) = 1$

$y'(x) = 4Ce^{4x} - 2De^{-2x} + 1$

$y(1) = 0$: $Ce^{4} + De^{-2} + 1 - 1 = 0$

$y'(1) = 1$: $\underbrace{4Ce^{4} - 2De^{-2} + 1 = 1}$

$Ce^{4} + De^{-2} = 0 \quad \cdot(2)$

$4Ce^{4} - 2De^{-2} = 0$

$6Ce^{4} = 0$

$C = 0$, $D = 0$

$\Rightarrow \underline{\underline{y(x) = x - 1}}$

10.6.6 $y'' + 2y' + 5y = \cos x$

homogene: $r^2 + 2r + 5 = 0 \Leftrightarrow r = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = \underbrace{-1}_{a} \pm \underbrace{2}_{b} i$

generell homogen løsning når røttene er $a \pm b$:

$$e^{ax}\left(C \cos \underline{b}x + D \sin \underline{b}x\right)$$

her: $e^{-x}\left(C\cos(2x) + D\sin(2x)\right)$

partikulære:

Vi prøver $y_p = A\cos x + B\sin x$

$y_p' = -A\sin x + B\cos x \qquad y_p'' = -A\cos x - B\sin x$

$y_p'' + 2y_p' + 5y_p = \underline{-A\cos x} - B\sin x - 2A\sin x + \underline{2B\cos x} + \underline{5A\cos x} + 5B\sin x$

$= (-A + 2B + 5A)\cos x + (-B - 2A + 5B)\sin x$

$= \underbrace{(4A + 2B)}_{1}\cos x + \underbrace{(4B - 2A)}_{0}\sin x$: $2A = 4B \Leftrightarrow A = 2B$

$4A + 2B = 8B + 2B = 1$

$B = \frac{1}{10}, \quad A = \frac{1}{5} \quad \Rightarrow \underline{y_p = \frac{1}{5}\cos x + \frac{1}{10}\sin x}$

generell løsning: $y = y_h + y_p = \underline{\underline{e^{-x}\left(C\cos(2x) + D\sin(2x)\right) + \frac{1}{5}\cos x + \frac{1}{10}\sin x}}$

12.2.2,

$\int_0^1 x^2 \, dx$ med midtpunktsmetoden med 2 delintervaller

$[0, \frac{1}{2}]$ og $[\frac{1}{2}, 1]$

midtpunkten: $\frac{1}{4}$ $\frac{3}{4}$ i boken

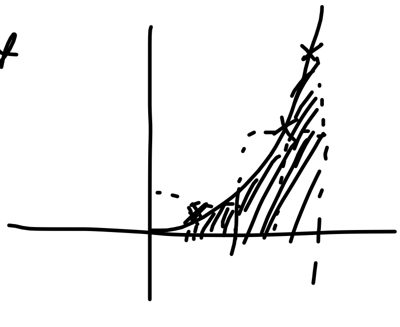
$f(\frac{1}{4}) \cdot h + f(\frac{3}{4}) \cdot h \quad \left(h \sum_{i=1}^{n} f(x_{i-\frac{1}{2}}) \right)$

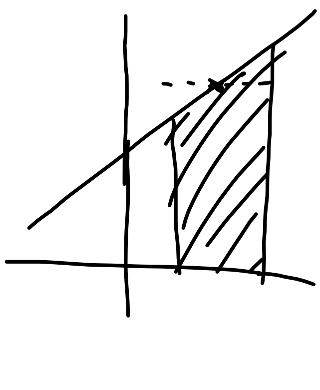
$\frac{1}{16} \cdot \frac{1}{2} + \frac{9}{16} \cdot \frac{1}{2} = \frac{1}{2}\left(\frac{1}{16} + \frac{9}{16}\right) = \frac{1}{2}\left(\frac{10}{16}\right)$

$= \frac{5}{16}$

det første svaralternativet

$\int_0^1 x^2 dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$





10.6.7

$y'' - 8y' + 6y = x^2$

$r^2 - 8r + 6 = 0 \qquad r = \dfrac{8 \pm \sqrt{64 - 4 \cdot 6}}{2} = \dfrac{8 \pm \sqrt{40}}{2} = \dfrac{8 \pm 2\sqrt{10}}{2} = 4 \pm \sqrt{10}$

homogen løsning: $Ce^{(4+\sqrt{10})x} + De^{(4-\sqrt{10})x}$

partikulær løsning: prøv $y_p = Ax^2 + Bx + C$

$y_p'' = 2A$

$y_p' = 2Ax + B$

$y_p'' - 8y_p' + 6y_p = 2A - 16Ax - 8B + 6Ax^2 + 6Bx + 6C$

$= 6Ax^2 + (-16A + 6B)x + 2A - 8B + 6C$

$= x^2 \Rightarrow A = \dfrac{1}{6}$

$-16A + 6B = 0 \Rightarrow -\dfrac{16}{6} + 6B = 0 \Rightarrow B = \dfrac{16}{36} = \underline{\underline{\dfrac{4}{9}}}$

$2A - 8B + 6C = 0$

$\dfrac{2}{6} - \dfrac{8 \cdot 4}{9} + 6C = 0$

$\dfrac{1}{3} - \dfrac{32}{9} + 6C = 0$

$6C = \dfrac{32 - 3}{9} = \dfrac{29}{9} \Rightarrow C = \underline{\underline{\dfrac{29}{54}}}$

generell løsning: $y = y_p + y_h = \underline{\underline{\dfrac{1}{6}x^2 + \dfrac{4}{9}x + \dfrac{29}{54} + Ce^{(4+\sqrt{10})x} + De^{(4-\sqrt{10})x}}}$