Kapittel 10 nullpunktsmetoder: Halveringsmetoden
Sekantmetoden
Newtons metode

26) $f(x) = \cos x$ på $[0,10]$

$f(0) = 1$

$f(10) = -0{,}8399 < 0$

$\left.\right\}$ nullpunkter til $\cos x$: $\underbrace{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2},}_{\text{ereste nullpunkter i } [0,10]} \frac{7\pi}{2}, \ldots$

midtpunkt: $\frac{0+10}{2} = 5$ $f(5) = \cos 5 \approx 0{,}28$

f skifter fortegn på $[5,10]$, ikke i $[0,5]$

halveringsmetoden velger derfor $[5,10]$

eneste nullpunktet til $\cos x$ i $[5,10]$ er $\frac{5\pi}{2}$, slik at halveringsmetoden vil finne denne.

neste steg ville blitt: midtpunkt $\frac{5+10}{2} = 7.5$

$f(7.5) = 0.35$

halveringsmetoden vil nå velge $[7.5, 10]$, osv.

10.3.2 Sekantmetoden: $x_i = x_{i-1} - \frac{x_{i-1} - x_{i-2}}{f(x_{i-1}) - f(x_{i-2})} f(x_{i-1})$

trenger to startpunkter x_0, x_1

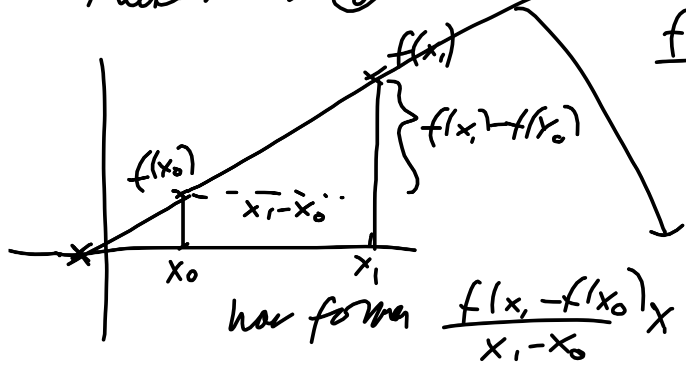
$f(x) = x^3 - 2$ $\quad x_0 = -2 \quad x_1 = 2$

$f(x_0) = -10 \quad f(x_1) = 6$

$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 2 - \frac{2-(-2)}{6-(-10)} 6 = 2 - \frac{4}{16} 6$

$= 2 - \frac{6}{4} = 2 - \frac{3}{2} = \underline{\underline{\frac{1}{2}}}$

→ alternativ d)



$\frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$\underline{\frac{f(x_1) - f(x_0)}{x_1 - x_0}} (x - x_0) + f(x_0)$

har formen $\frac{f(x_1) - f(x_0)}{x_1 - x_0} x + z$

$x = x_0 \quad f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \underline{x_0} + z \Rightarrow z = f(x_0) - \frac{f(x_1) - f(x_0)}{x_1 - x_0} x_1$

$\Rightarrow$ linjen $y = \underline{\frac{f(x_1) - f(x_0)}{x_1 - x_0} x} + f(x_0) - \underline{\frac{f(x_1) - f(x_0)}{x_1 - x_0} x_0}$

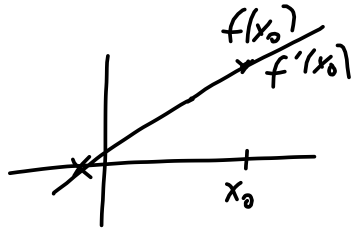
$y = \frac{f(x_1) - f(x_0)}{x_1 - x_0} (x - x_0) + f(x_0) = 0$ for hvilken x?

$(x - x_0) \frac{f(x_1) - f(x_0)}{x_1 - x_0} = -f(x_0)$

$x - x_0 = -\frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$

$x = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$

Newton:



$y = f'(x_0)(x - x_0) + f(x_0)$
$= 0$

$f'(x_0)(x - x_0) = -f(x_0)$

$x - x_0 = -\frac{f(x_0)}{f'(x_0)}$

$\underline{\underline{x = x_0 - \frac{f(x_0)}{f'(x_0)}}}$

13.4.2 $\quad x'(t) = \underbrace{x}_{f(t,x)} \quad x(0) = 1$

a) Euler med et steg for å finne $x(1)$. $h = 1 \quad \left(\frac{b-a}{n}\right)$

$$X_1 = X_0 + hf(t_0, x_0) \qquad f(t_0,x_0) = f(\underset{t}{0}, \underset{x}{1}) = 1$$

$$= 1 + 1 \cdot 1 = \underline{\underline{2}}$$

b) et steg med Euler midtpunkt:

$$X_{\frac{1}{2}} = X_0 + \frac{h}{2} f(t_0, x_0) = 1 + \frac{1}{2} \cdot 1 = \frac{3}{2}$$

$$X_1 = X_0 + hf(\underbrace{t_{\frac{1}{2}}, x_{\frac{1}{2}}}) = 1 + 1 \cdot X_{\frac{1}{2}} = 1 + 1 \cdot \frac{3}{2} = \underline{\underline{\frac{5}{2}}}$$

$x' = x \qquad x(0) = 1$

$\frac{x'}{x} = 1$

$0 = 0 + C$

$\ln|x| = t + C \qquad x = 1, \; t = 0 \Rightarrow C = 0$

$\ln|x| = t \qquad |x| = e^t, \qquad x(t) = e^t$ siden $x(0) = 1$

$x(1) = e^1 = e \approx 2.71828\ldots$

To steg med Euler for å tilnærme $x(1)$. $h = \frac{1}{2}$

$$X_1 = X_0 + hf(t_0, x_0) = 1 + \frac{1}{2} X_0 = \frac{3}{2}$$

$$X_2 = X_1 + hf(t_1, x_1) = \frac{3}{2} + \frac{1}{2} X_1 = \frac{3}{2} + \frac{1}{2} \cdot \frac{3}{2}$$

$$= \frac{3}{2} + \frac{3}{4}$$

$$= \frac{6+3}{4} = \frac{9}{4}$$

$$= 2.25$$