Mock mid-term exam
MAT-INF 1100, Autmn 2003

This mock mid-term exam has the same format as the real mid-term exam, and consists of problems of the same type and level of difficulty. The first 15 problems count two points each while the last 5 count 4 points each. The maximum total is therefore 50 points. There are 5 alternative answers for each problem, but only one is correct. If your answer is wrong or you do not tick off one of the answers you get zero points for this problem. In other words, you will not be penalised with negative points for answering incorrectly.

Problem and answer sheets

1) The binary number 1010101 is the same as the decimal number

☐ 12

☐ 67

☐ 54

☐ 85

☐ 91

2) Written in binary the number 183 becomes

☐ 10110111

☐ −1010101

☐ 11100

☐ 10110011

☐ 11001111

3) The real number $1 + \sqrt{2}$ is

☐ $1 - \sqrt{2}$

☐ a rational number

☐ a natural number

☐ not defined

☐ an irrational number

4) The real number $\dfrac{6}{7\sqrt{7} - 7} - \dfrac{1}{\sqrt{7}}$ is

- ☐ an irrational number
- ☐ a negative number
- ☐ 7
- ☐ 0
- ☐ a rational number

5) The least upper bound of the set $\{x : |x-2| < 2\}$ is

- ☐ 0
- ☐ 2
- ☐ $\sqrt{2}$
- ☐ 4
- ☐ -2

6) The least upper bound of the set $\{x : x^2 + 2 < 3x\}$ is

- ☐ 3
- ☐ 0
- ☐ 2
- ☐ $\sqrt{3}$
- ☐ 1

7) Suppose that we multiply together the factors in the expression $(a+1)^{19}$, where a is different from 0, what will the coefficient multiplying a^{17} be?

- ☐ 17
- ☐ 136
- ☐ 153
- ☐ 171
- ☐ 19

8) Which of the following expressions may give a large relative error for certain values of a and b when the computations are done with floating-point numbers?

- ☐ $a+b$
- ☐ a/b
- ☐ $a*b$
- ☐ $2a/b$
- ☐ $\sqrt{ab}$

9) Which one of the following statements are true?

☐ There is a real number that is greater than all natural numbers

☐ Any real number can be approximated arbitrarily well by a rational number

☐ Round-off errors can never become a problem on a calculator

☐ There are only a finite number of irrational numbers

☐ There are infinitely many 64-bit floating-point numbers

10) The function f is defined on the interval $[1, 2]$, it is continuous, and satisfies the condition $f(1) \cdot f(2) < 0$. After 9 iterations with the bisection method we will have an estimate for one of the zeros of f absolute error less than

☐ -0.3

☐ 0.002

☐ 0.0001

☐ 10^{-5}

☐ e^{-15}

11) Which one of the following difference equations is linear?

☐ $x_n - \sin(x_{n-1}) + x_{n-2} = 0$

☐ $e^{x_n} + x_{n-1} = 0$

☐ $x_n + x_{n-1}x_{n-2} = 0$

☐ $x_n + n\,x_{n-1} + x_{n-2} = 0$

☐ $\sqrt{x_n} - x_{n-1} = 0$

12) The solution of the difference equation

$$x_{n+2} + x_{n+1} - 2x_n = 0, \qquad x_0 = 1, \quad x_1 = 1$$

is given by

☐ $x_n = (3 \cdot 2^n - 4^n)/2$

☐ $x_n = 0$

☐ $x_n = 2^{n+1} - 3^n$

☐ $x_n = 4n$

☐ $x_n = 1$

13) A difference equation has characteristic equation with roots $r_1 = 1 + i$ and $r_2 = 1 - i$. The difference equation is then given by

- ☐ $x_{n+2} - 2x_{n+1} + 2x_n = 0$
- ☐ $x_{n+2} - x_n = 0$
- ☐ $x_{n+2} + x_{n+1} - x_n = 0$
- ☐ $x_{n+2} - 4x_{n+1} + 4x_n = 0$
- ☐ $x_{n+2} - 8x_{n+1} + x_n = 0$

14) The solution of the difference equation

$$x_{n+2} - 6x_{n+1} + 9x_n = 0, \qquad x_0 = 0, \quad x_1 = 3$$

is given by

- ☐ $x_n = 4^n - 1$
- ☐ $x_n = n\,3^n$
- ☐ $x_n = 3\,(3^n - 1)/2$
- ☐ $x_n = 3n$
- ☐ $x_n = 7n - 4$

15) Numerical simulation of the difference equation $x_{n+2} - 4x_{n+1} + x_n = 0$ with initial values $x_0 = 1$ and $x_1 = 4$ will result in

- ☐ major problems with round-off errors
- ☐ no problems with round-off errors
- ☐ $x_n = (n+1)^2$
- ☐ a solution that is always 0
- ☐ $x_2 = 3$

16) We let P_n denote the statement that the formula

$$1 + 2 + \cdots + n = \frac{1}{8}(2n+1)^2$$

is true. To prove this by induction we may proceed as follows:

1. We easily see that P_1 is true.
2. Suppose we have proved that $P_1, \ldots, P_k$ is true, to complete the proof we must prove that then P_{k+1} is also true. Since P_k is true we have

$$\begin{aligned} 1 + 2 + \cdots + k + (k+1) &= \frac{1}{8}(2k+1)^2 + k + 1 = \frac{1}{2}k^2 + \frac{3}{2}k + \frac{9}{8} \\ &= \frac{1}{8}(4k^2 + 12k + 9) \\ &= \frac{1}{8}(2k+3)^2 = \frac{1}{8}\bigl(2(k+1)+1\bigr)^2. \end{aligned}$$

In other words, if P_k is true, then P_{k+1} must also be true.

Which one of the following statements are true?

- ☐ The statement P_n is true, but part 2 of the proof is wrong
- ☐ The statement P_n is wrong and part 1 of the proof is wrong
- ☐ The statement P_n is wrong, and both part 1 and part 2 of the proof is wrong
- ☐ Both the statement P_n and the proof are correct
- ☐ The proof is correct, but it is not a proof by induction

17) The difference equation

$$x_n = x_{n-1} + x_{n-2}^2, \qquad \text{der } x_0 = 0 \text{ og } x_1 = 1$$

is given. We believe that the following statement is true:

P_n: For all integers $n \geq 0$ it holds that x_{3n} is an even number while x_{3n+1} and x_{3n+2} are both odd numbers.

We try to prove this by induction:

1. For $n = 0$ we have $x_{3n} = x_0 = 0$ which is even, while $x_{3n+1} = x_1 = 1$ which is odd. We also have $x_{3n+2} = x_2 = x_1 + x_0^2 = 1$ which is even as well, so P_n is true for $n = 0$.
2. Suppose that we have shown P_n to be true for $n = 0, \ldots, k-1$; to complete the proof we have to show that then P_k is also true. We have $x_{3k} = x_{3k-1} + x_{3k-2}^2$, and from the inductive hypothesis we know that x_{3k-2} and x_{3k-1} are both odd numbers. then x_{3k-2}^2 is also odd, and since the sum of two odd numbers is even it is clear that x_{3k} is even. In the same way we have $x_{3k+1} = x_{3k} + x_{3k-1}^2$. We now know that x_{3k} is an even number while x_{3k-1}^2 is odd. But then x_{3k+1} is odd. Finally, we must check x_{3k+2}. We have $x_{3k+2} = x_{3k+1} + x_{3k}^2$, and from what we have just shown it is clear that x_{3k+1} is odd while x_{3k} is an even number. Then x_{3k}^2 is also even so x_{3k+2} is the sum of an even and an odd number and therefore an odd number.

Which one of the following statements are true?

- ☐ The statement P_n is true, but part 1 of the proof is wrong
- ☐ The statement P_n is true, but part 2 of the proof is wrong
- ☐ The statement P_n is wrong, but the proof is correct
- ☐ Both the statement P_n and the proof are correct
- ☐ The proof is correct, but it is not a proof by induction

18) The solution of the inhomogenous difference equation

$$x_{n+2} - 4x_{n+1} + 4x_n = n$$

where $x_0 = 1$ and $x_1 = 1$ is given by

- ☐ $x_n = 2^n - n$
- ☐ $x_n = n$
- ☐ $x_n = (n^2 + n)/2$
- ☐ $x_n = n + 2 - 2^n$
- ☐ $x_n = n\,2^n - 2n + 1$

19) The solution of the inhomogenous difference equation

$$x_{n+2} + x_n = n^2$$

where $x_0 = 1$ and $x_1 = 0$ is given by

- ☐ $x_n = n - 1$
- ☐ $x_n = n^2 - 1$
- ☐ $x_n = \frac{1}{2}n(n-2) + \cos(n\pi/2) + \frac{1}{2}\sin(n\pi/2)$
- ☐ $x_n = n^2 + \sin(n\pi/2) + \cos(n\pi/2)$
- ☐ $x_n = \cos(n\pi/2)$

20) We use the bisection method on a function f defined on the interval $[0, 1]$. The function is such that when we choose the next interval we alternatively choose the left and right subintervals. If we denote the seuence of midpoints by (m_i) this means that we first have $m_1 = 1/2$, then we move to the left so that $m_2 = 1/4$, next time we move to the right with $m_3 = 3/8$, next time to the left with $m_4 = 5/16$, next time to the right and so on.

The midpoints can be characterised by the difference equation

- ☐ $m_n = m_{n-1}/2, \qquad m_0 = 1$
- ☐ $m_n = \frac{1}{2}(m_{n-1} + m_{n-3}), \qquad m_{-1} = 0, \quad m_0 = 1$
- ☐ $m_n = \frac{1}{2}(m_{n-1} + m_{n-2}), \qquad m_{-1} = 1, \quad m_0 = 0$
- ☐ $m_n = \frac{1}{2}(m_n + m_{n-1}), \qquad m_0 = 1$
- ☐ $m_n = \frac{1}{2}(1 + m_{n-1}), \qquad m_0 = 1$

The end!!