

A Computational Perspective on Calculus

Knut Mørken

November 23, 2007

Contents

1	Introduction	1
1.1	A bit of history	1
1.2	Computers and different types of information	3
1.2.1	Text	4
1.2.2	Sound	4
1.2.3	Images	4
1.2.4	Film	4
1.2.5	Geometric form	5
1.2.6	Laws of nature	5
1.2.7	Virtual worlds	5
1.2.8	Summary	6
1.3	Computation by hand and by computer	7
1.4	Algorithms and programs	8
1.5	Doing computations on a computer	10
1.5.1	How can computers be used for calculations?	10
1.5.2	What do you need to know?	11
1.5.3	Different computing environments	12
2	0 and 1	15
2.1	Robust communication	15
2.2	Why 0 and 1 in computers?	15
2.3	True and False	17
2.3.1	Logical variables and logical operators	18
2.3.2	Combinations of logical operators	20
2.4	Set theory	21
3	Numbers and Numeral Systems	23
3.1	Terminology and Notation	23
3.2	Natural Numbers in Different Numeral Systems	25
3.2.1	Alternative Numeral Systems	25

3.2.2	Conversion to the Base- β Numeral System	28
3.3	Representation of Fractional Numbers	31
3.3.1	Rational and Irrational Numbers in Base- β	31
3.3.2	An Algorithm for Converting Fractional Numbers	35
3.3.3	Properties of Fractional Numbers in Base- β	36
3.4	Arithmetic in Base β	40
3.4.1	Addition	40
3.4.2	Subtraction	41
3.4.3	Multiplication	41
4	Computers, Numbers and Text	45
4.1	Representation of Integers	45
4.1.1	Bits, bytes and numbers	46
4.1.2	Fixed size integers	47
4.1.3	Integers in Java	48
4.1.4	Integers in Python	49
4.1.5	Division by zero	49
4.2	Computers and real numbers	49
4.2.1	Representation of real numbers	50
4.2.2	Floating point numbers in Java	54
4.2.3	Floating point numbers in Python	54
4.3	Representation of letters and other characters	55
4.3.1	The ASCII table	56
4.3.2	ISO latin character sets	57
4.3.3	Unicode	57
4.3.4	UTF-8	58
4.3.5	UTF-16	60
4.3.6	UTF-32	62
4.3.7	Text in Java	62
4.3.8	Text in Python	62
4.4	Representation of general information	62
4.4.1	Text	62
4.4.2	Numbers	63
4.4.3	General information	64
4.4.4	Computer programs	64
4.5	A fundamental principle of computing	64

5	Lossless Compression	71
5.1	Introduction	72
5.1.1	Run-length coding	73
5.2	Huffman coding	73
5.2.1	Binary trees	75
5.2.2	Huffman trees	76
5.2.3	The Huffman algorithm	77
5.2.4	Properties of Huffman trees	81
5.3	Probabilities and information entropy	82
5.3.1	Probabilities rather than frequencies	82
5.3.2	Information entropy	84
5.4	Arithmetic coding	85
5.4.1	Arithmetic coding basics	85
5.4.2	An algorithm for arithmetic coding	87
5.4.3	Properties of arithmetic coding	91
5.4.4	A decoding algorithm	93
5.4.5	Arithmetic coding in practice	95
5.5	Lempel-Ziv-Welch algorithm	96
5.6	Lossless compression programs	96
5.6.1	Compress	97
5.6.2	gzip	97
6	Audio compression in practice	101
6.1	Wavelet based compression	101
6.2	Fourier analysis and the DCT	103
6.2.1	The Discrete Cosine Transform	105
6.3	Psycho-acoustic models	109
6.4	Digital audio formats	110
6.4.1	Audio sampling—PCM	110
6.4.2	Lossless formats	111
6.4.3	Lossy formats	112
7	Functions of two variables	115
7.1	Basics	115
7.1.1	Basic definitions	115
7.1.2	Differentiation	117
7.1.3	Vector functions of several variables	119
7.2	Numerical differentiation	120

8	Digital images and image formats	125
8.1	What is an image?	125
8.1.1	Light	125
8.1.2	Digital output media	126
8.1.3	Digital input media	127
8.1.4	Definition of digital image	127
8.1.5	Images as surfaces	129
8.2	Operations on images	131
8.2.1	Normalising the intensities	131
8.2.2	Extracting the different colours	132
8.2.3	Converting from colour to grey-level	132
8.2.4	Computing the negative image	134
8.2.5	Increasing the contrast	134
8.2.6	Smoothing of an image	137
8.2.7	Detecting edges	138
8.2.8	Comparing the first derivatives	142
8.2.9	Second-order derivatives	142
8.3	Image formats	144
8.3.1	Raster graphics and vector graphics	144
8.3.2	Vector graphics formats	146
8.3.3	Raster graphics formats	147