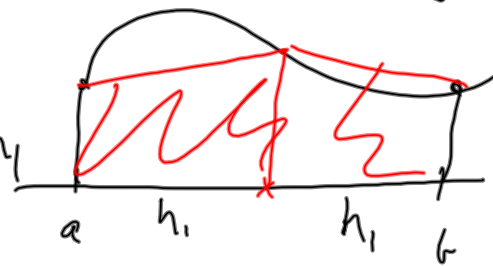


$$\bar{I}_b = \frac{f(a) + f(b)}{2} (b - a)$$

$$I_i = \frac{f(a) + f(a+h_i)}{2} \cdot h_i + \frac{f(a+h_i) + f(b)}{2} \cdot h_i$$

$$= \frac{f(a) + f(b)}{2} \cdot h_i + f(a+h_i) \cdot h_i$$



$$x' = f(t, x) = 1 + x^2$$

$$f\left(t_k + \frac{h}{2}, \frac{x_k + x_{k+1}}{2}\right) = 1 + \left(\frac{x_k + x_{k+1}}{2}\right)^2$$

Ex. 16.4.4 i Kalkulus.

Vi har en dyrenpopulasjon som vokser med vekstrate  $r$  - urealistisk.

Mer rimelig å si at det fins et maksimalt antall individer  $N$  som kan leve med de gitte ressursene. Kan beskrives ved

$$y'(t) = r \cdot y(t) \left(1 - \frac{y(t)}{N}\right), \quad y(0) = y_0$$

Hvis  $y(t) \neq 0$  og  $y(t) \neq N$  har vi

$$\frac{y'(t)}{y(t)(1-y(t)/N)} = r$$

Integrerer på begge sider

$$\int \frac{y'(t) dt}{y(t)(1-y(t)/N)} = \int r dt = r \cdot t + C$$

$$\int \frac{dy}{y(1-y/N)} \quad \left| \quad \frac{1}{y(1-y/N)} = \frac{1}{y} + \frac{1/N}{1-y/N} \right.$$

$$= \int \frac{dy}{y} + \int \frac{dy}{N-y}$$

$$= \ln|y| - \ln|N-y| = \ln \left| \frac{y}{N-y} \right| = \frac{r \cdot t}{y(N-y)}$$

$$\ln \left| \frac{y}{N-y} \right| = r \cdot t + C$$

$$e^{\ln \left| \frac{y}{N-y} \right|} = e^{r \cdot t + C}$$

$$\left| \frac{y}{N-y} \right| = e^C \cdot e^{rt}$$

$$\frac{y}{N-y} = \pm e^C \cdot e^{rt}$$

$$\frac{y}{N-y} = D \cdot e^{rt}, \quad D \in \mathbb{R}, D \neq 0$$

$$y = (N-y) D e^{rt} = N \cdot D e^{rt} - y \cdot D e^{rt}$$

$$y = \frac{D \cdot N e^{rt}}{1 + D e^{rt}} = \frac{N}{1 + e^{-rt}/D}$$

$$y + y \cdot D e^{rt} = N \cdot D e^{rt}$$

$$y(1 + D e^{rt}) = N \cdot D e^{rt}$$

$$y = \frac{N \cdot D e^{rt}}{1 + D e^{rt}}$$