Numerical Algorithms
and
Digital Representation

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Preface

These lecture notes form part of the syllabus for the first-semester course MAT-INF1100 at the University of Oslo. The topics roughly cover two main areas: *Numerical algorithms*, and what can be termed *digital understanding*. Together with a thorough understanding of calculus and programming, this is knowledge that students in the mathematical sciences should gain as early as possible in their university career. As subjects such as physics, meteorology and statistics, as well as many parts of mathematics, become increasingly dependent on computer calculations, this training is essential.

Our aim is to train students who should not only be able *to use* a computer for mathematical calculations; they should also have a basic understanding of *how* the computational methods work. Such understanding is essential both in order to judge the quality of computational results, and in order to develop new computational methods when the need arises.

In these notes we cover the basic numerical algorithms such as interpolation, numerical root finding, differentiation and integration, as well as numerical solution of ordinary differential equations. In the area of digital understanding we discuss digital representation of numbers, text, sound and images. In particular, the basics of lossless compression algorithms with Huffman coding and arithmetic coding is included.

A basic assumption throughout the notes is that the reader either has attended a basic calculus course in advance or is attending such a course while studying this material. Basic familiarity with programming is also assumed. However, I have tried to quote theorems and other results on which the presentation rests. Provided you have an interest and curiosity in mathematics, it should therefore not be difficult to read most of the material with a good mathematics background from secondary school.

MAT-INF1100 is a central course in the project *Computers in Science Edu-*

cation (CSE) at the University of Oslo. The aim of this project is to make sure that students in the mathematical sciences get a unified introduction to computational methods as part of their undergraduate studies. The basic foundation is laid in the first semester with the calculus course, MAT1100, and the programming course INF1100, together with MAT-INF1100. The maths courses that follow continue in the same direction and discuss a number of numerical algorithms in linear algebra and related areas, as well as applications such as image compression and ranking of web pages.

Some fear that a thorough introduction of computational techniques in the mathematics curriculum will reduce the students' basic mathematical abilities. This could easily be true if the use of computations only amounted to running code written by others. However, deriving the central algorithms, programming them, and studying their convergence properties, should lead to a level of mathematical understanding that should certainly match that of a more traditional approach.

Many people have helped develop these notes which have matured over a period of ten years. Øyvind Ryan, Andreas Våvang Solbrå, Solveig Bruvoll, and Marit Sandstad have helped directly with recent versions, while Pål Hermunn Johansen provided extensive programming help with an earlier version. For this latest version, Andreas Våvang Solbrå has provided important assistance and feedback. Geir Pedersen was my co-lecturer for four years. He was an extremely good discussion partner on all the facets of this material, and influenced the list of contents in several ways. I work at the Centre of Mathematics for Applications (CMA) at the University of Oslo, and I am grateful to the director, Ragnar Winther, for his enthusiastic support of the CSE project and my extensive undertakings in teaching. Over many years, my closest colleagues Geir Dahl, Michael Floater, and Tom Lyche have shaped my understanding of numerical analysis and allowed me to spend considerably more time than usual on elementary teaching. Another colleague, Sverre Holm, has been my source of information on signal processing. To all of you: thank you!

My academic home, the Department of Informatics and its chairman Morten Dæhlen, has been very supportive of this work by giving me the freedom to extend the Department's teaching duties, and by extensive support of the CSE-project. It has been a pleasure to work with the Department of Mathematics over the past eight years, and I have many times been amazed by how much confidence they seem to have in me. I have learnt a lot, and have thoroughly enjoyed teaching at the cross-section between mathematics and computing which is my scientific home. I can only say thank you, and I feel at home in both departments.

A course like MAT-INF1100 is completely dependent on support from other

courses. Tom Lindstrøm has done a tremendous job with the parallel calculus course MAT1100, and its sequel MAT1110 on multivariate analysis and linear algebra. Hans Petter Langtangen has done an equally impressive job with INF1100, the introductory programming course with a mathematical and scientific flavour, and I have benefited from many hours of discussions with both of them. Morten Hjorth-Jensen, Arnt-Inge Vistnes and Anders Malthe-Sørenssen with colleagues have introduced a computational perspective in a number of physics courses, and discussions with them have convinced me of the importance of introducing computations for all students in the mathematical sciences. Thank you to all of you.

The CSE project is run by a group of people: Morten Hjorth-Jensen and Anders Malthe-Sørenssen from the Physics Department, Hans Petter Langtangen from the Simula Research Lab and the Department of Informatics, Øyvind Ryan from the CMA, Annik Myhre (Dean of Education at the MN-faculty[1]), Hanne Sølna (Head of the Studies section at the MN-faculty[1]), Helge Galdal (Administrative Leader of the CMA), and myself. This group of people has been the main source of inspiration for this work, and without you, there would still only be uncoordinated attempts at including computations in our elementary courses. Thank you for all the fun we have had.

The CSE project has become much more than I could ever imagine, and the reason is that there seems to be a genuine collegial atmosphere at the University of Oslo in the mathematical sciences. This means that it has been possible to build momentum in a common direction not only within a research group, but across several departments, which seems to be quite unusual in the academic world. Everybody involved in the CSE project is responsible for this, and I can only thank you all.

Finally, as in all teaching endeavours, the main source of inspiration is the students, without whom there would be no teaching. Many students become frustrated when their understanding of the nature of mathematics is challenged, but the joy of seeing the excitement in their eyes when they understand something new is a constant source of satisfaction.

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[1] The Faculty of Mathematics and Natural Sciences.

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