11.2.9 kalkulus

Vi skal finne $\int_0^1 \frac{1-e^{-t}}{t}\,dt$, med nøyaktighet på 10^{-3}.

Ikke regn ut Taylorrekken til integranden direkte. Bruk heller

$$e^x = T_n(x) + R_n(x) = 1 + x + \dots + \frac{x^n}{n!} + \frac{e^c}{(n+1)!}x^{n+1} \quad (c \text{ mellom } 0 \text{ og } x)$$

sett inn $x = -t$

$$\frac{1-e^{-t}}{t} = \frac{1 - (1 + (-t) + \dots + \frac{(-t)^n}{n!} + \frac{e^{c(t)}}{(n+1)!}(-t)^{n+1})}{t}$$

$$= \frac{t + \dots (-1)^{n+1}\frac{t^n}{n!} + (-1)^{n+2}\frac{e^c}{(n+1)!}t^{n+1}}{t}$$

$$= 1 + \dots + (-1)^{n+1}\frac{t^{n-1}}{n!} + (-1)^{n+2}\frac{e^c}{(n+1)!}t^n$$

$$\int_0^1 \frac{1-e^{-t}}{t}\,dt = \underbrace{\int_0^1 \left(1 + \dots (-1)^{n+1}\frac{t^{n-1}}{n!}\right)dt}_{\text{vår approksimasjon}} + \underbrace{\int_0^1 (-1)^{n+2}\frac{e^c}{(n+1)!}t^n\,dt}_{\text{rest, vil ha mindre enn } 10^{-3}}$$

$$\left|\int_0^1 (-1)^{n+2}\frac{e^c}{(n+1)!}t^n\,dt\right| \underset{NB}{\leq} \int_0^1 \left|(-1)^{n+2}\frac{e^c}{(n+1)!}t^n\right|dt = \int_0^1 \left|\frac{e^c t^n}{(n+1)!}\right|dt$$

$$\leq \int_0^1 \frac{t^n}{(n+1)!}\,dt = \left[\frac{t^{n+1}}{(n+1)(n+1)!}\right]_0^1 = \frac{1}{(n+1)(n+1)!}$$

c er mellom 0 og $-t$ $\Rightarrow e^c \leq 1$

vi må velge n slik at $\frac{1}{(n+1)(n+1)!} \leq 10^{-3}$. Dette blir $n=5$

$\Rightarrow$ approksimasjonen blir

$$\int_0^1 \left(1 + \dots (-1)^{n+1}\frac{t^{n-1}}{n!}\right)dt = \int_0^1 \left(1 - \frac{t}{2} + \frac{t^2}{6} - \frac{t^3}{24} + \frac{t^4}{120}\right)dt$$

$$= \dots \frac{5737}{7200} \approx \underline{\underline{0.7698}}$$

Oppgave 9.2.4 (kompendiet)

a) Vi antar $p_1(x_0) = p_2(x_0)$, $p_1(x_1) = p_2(x_1)$, $p_1(x_2) = p_2(x_2)$

Vi setter $p = p_2 - p_1$. p er null i x_0, x_1, x_2, siden

$p(x_0) = p_2(x_0) - p_1(x_0) = 0$

$p(x_1) = p_2(x_1) - p_1(x_1) = 0$

$p(x_2) = p_2(x_2) - p_1(x_2) = 0$

b) siden $\underset{\parallel\; ax^2+bx+c}{p_2 - p_1}$ er 0 i x_0, x_1, x_2:

kan dele på $\underset{\neq 0}{x_0 - x_1}$, $\underset{\neq 0}{x_0 - x_2}$

$ax_0^2 + bx_0 + c = 0$

$ax_1^2 + bx_1 + c = 0$

$ax_2^2 + bx_2 + c = 0$

$\Rightarrow a(\underbrace{x_0^2 - x_1^2}_{(x_0-x_1)(x_0+x_1)}) + b(x_0 - x_1) = 0$

$a(x_0^2 - x_2^2) + b(x_0 - x_2) = 0$

$\Rightarrow a(x_0 + x_1) + b = 0$

$a(x_0 + x_2) + b = 0$

$\Rightarrow a(x_1 - x_2) = 0 \Rightarrow a = 0$ (del med $x_1 - x_2 \neq 0$)

$\Rightarrow b = 0 \Rightarrow c = 0$

$\Rightarrow p = 0$, så $p_1 = p_2$

9.3.3

x	x_0: 0	x_1: 1	x_2: 2
$f(x)$	2	1	0

Newtonformen til interpolerende polynom:

$$p_2(x) = \underset{\substack{\parallel \\ 2}}{f(x_0)} + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$f[x_0,x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{1-2}{1-0} = -1$$

$$f[x_1,x_2] = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{0-1}{2-1} = -1$$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{-1-(-1)}{2-0} = 0$$

$$\Rightarrow p_2(x) = 2-(x-0)+0\cdot(x-0)(x-1) = 2-x$$

i oppgaven står det at tabellen ga samplene til $f(x)=2-x$

$\Rightarrow$ her er $f(x)=p_2(x)$, som vi viste i oppgave 9.2.4

10.4.7 $\quad f(x) = \frac{1}{x} - R \longrightarrow x = \frac{1}{R} \qquad f'(x) = -\frac{1}{x^2}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\frac{1}{x_n} - R}{-\frac{1}{x_n^2}}$$

$$= x_n + x_n^2\left(\frac{1}{x_n} - R\right) = x_n + x_n - R x_n^2$$

$$= 2x_n - R x_n^2 = \underline{x_n(2 - R x_n)}$$