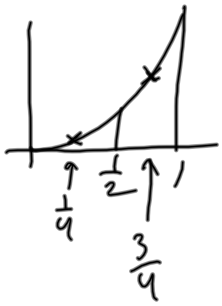
Kompendiet

12.2.2: midtpunktsmetoden til å tilnærme $\int_0^1 x^2 dx$ (to intervaller):



$$I_{mid} = h\left(f\left(\tfrac{1}{4}\right) + \left(\tfrac{3}{4}\right)\right) = \frac{1}{2}\left(\frac{1}{16}+\frac{9}{16}\right) = \frac{1}{2}\cdot\frac{10}{16} = \frac{5}{16}$$

(alternativ 1)

12.3.2: Trapesmetoden på samme:

$$I_{trap} = h\left(\frac{f(0)+f(1)}{2} + f\left(\tfrac{1}{2}\right)\right) = \frac{1}{2}\left(\frac{0+1}{2}+\frac{1}{4}\right) = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{4}\right) = \frac{3}{8}$$

(feil i fasiten)

Kalkulus

10.2.1

Vekst per år : $0{,}02\,y(t)$ (2% økning)

Tilskudd pga innvandring : 40 000

Vi får da : $\underline{y'(t) = 0.02\,y(t) + 40000}$

$$\Rightarrow y'(t) \underbrace{-\,0.02\,y(t)}_{f(t)} = \underbrace{40000}_{g(t)}$$

$$\Rightarrow F(t) = -0.02t. \text{ Integrerende faktor} = e^{F(t)} = e^{-0.02t}$$

$$\Rightarrow y'(t)e^{-0{,}02t} - 0.02\,y(t)e^{-0.02t} = 40000\,e^{-0.02t}$$

$$\Rightarrow \left(y(t)e^{-0.02t}\right)' = 40000e^{-0.02t}$$

$$y(t)e^{-0.02t} = \frac{40000}{-0.02}e^{-0{,}02t} + C$$

$$\underline{y(t) = -2\,000\,000 + C\,e^{0.02t}} \quad \text{generell løsning.}$$

Her $y(0) = 2\,000\,000 \Rightarrow 2\,000\,000 = -2\,000\,000 + C$

$$\Rightarrow C = \underline{4\,000\,000}$$

$$\Rightarrow \underline{\underline{y(t) = -2\,000\,000 + 4\,000\,000\,e^{0.02t}}}$$

10.2.10

$y(t)$ = antall liter klor som finnes i badevannet.

Vi mister $\underline{\frac{1}{20}y(t)}$ l. klor per døgn, siden 50 000 er $\frac{1}{20}$ av 1 000 000

Vi få hvert døgn $\frac{0.001}{100} \times 50000 = \underline{0.5}$ l. klor. → klor inn

Derfor: $y'(t) = -\frac{1}{20}y(t) + 0.5$

$$\Rightarrow \underline{\underbrace{y'(t) + \tfrac{1}{20}}_{f(t)=\frac{1}{20}=0.05} y(t) = \underbrace{0.5}_{g}} \qquad e^{F(t)} = e^{0.05t}$$

Setning 10.1.3: $y(t) = e^{-0.05t}\left(\int \frac{1}{2}e^{0.05t} + C\right)$

$$= e^{-0.05t}\left(10e^{0.05t} + C\right) = \underline{\underline{10 + Ce^{-0.05t}}}$$

initialkrav: $y(0) = \frac{0.004}{100} \cdot 1000000 = \underline{40}$

setter inn: $40 = 10 + C \Rightarrow C = 30 \Rightarrow \underline{\underline{y(t) = 10 + 30e^{-0.05t}}}$

Når er klorprosent 0.003?

$$\frac{0.003}{100} \times 1000000 = 30l$$

$$\Rightarrow 10 + 30e^{-0.05t} = 30 \Rightarrow 3e^{-0.05t} = 2 \Rightarrow e^{-0.05t} = \frac{2}{3}$$

$$\Rightarrow -0.05t = \ln\frac{2}{3} = \ln 2 - \ln 3 \Rightarrow t = 20(\ln 3 - \ln 2) \approx \underline{\underline{8.1093}}$$

10.4

1d) $xyy' = 1+x^2+y^2+x^2y^2$
$= (1+x^2)+y^2(1+x^2) = (1+x^2)(1+y^2)$

$\frac{1}{2} \frac{2yy'}{1+y^2} = \frac{1+x^2}{x} = \frac{1}{x}+x$

$y'(x)dx = dy$
$u = 1+y^2$

$\frac{1}{2}\ln(1+y^2) = \ln|x| + \frac{1}{2}x^2 + C$
$\ln(1+y^2) = 2\ln|x| + x^2 + 2C$
$1+y^2 = e^{2\ln|x| + x^2 + 2C} = e^{\ln|x|^2} e^{x^2} D = D|x|^2 e^{x^2} = \underline{Dx^2 e^{x^2}}$

$\Rightarrow \underline{\underline{y(x) = \pm\sqrt{Dx^2 e^{x^2} - 1}}}$

10.4

2e) $y' + 2e^{y} = 0 \qquad y(0) = 0$

$$y' = -2e^{y}$$

$$e^{-y} y' = -2$$

$$-e^{-y} = -2x + C \Rightarrow e^{-y} = 2x - C \Rightarrow -y = \ln(2x-c)$$

$$\Rightarrow \underline{y(x) = -\ln(2x - C)} \qquad (\ln 1 = 0)$$

$$y(0) = 0 \Rightarrow \quad 0 = -\ln(0-c) = -\ln(\underbrace{-C}_{1}) \Rightarrow C = -1$$

$$\Rightarrow y(x) = \underline{\underline{-\ln(2x+1)}}$$