10.3.5

$f(x) = (x-1)^3$ $\quad x_0 = 0.5 \quad x_1 = 1.2$

$f'(x) = 3(x-1)^2$

eneste nullpkt: $x = 1$, $f'(1) = 0$

Da finnes ingen γ (se teo 10.14) slik at $|f'(x)| > \gamma > 0$ nor $x = 1$.

Får vi nå 62% flere viktige sifter per iterasjon?

11.3.4. Andreordens Taylor med restledd

(a) $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(\xi)}{6}(x-a)^3$

sett $x = a+h,\ a-h$

$$f(a+h) = f(a) + f'(a)h + \frac{f''(a)}{2}h^2 + \frac{f'''(\xi_1)}{6}h^3 \quad \xi_1 \in [a, a+h]$$

$$f(a-h) = f(a) - f'(a)h + \frac{f''(a)}{2}h^2 - \frac{f'''(\xi_2)}{6}h^3 \quad \xi_2 \in [a-h, a]$$

trekk disse fra hverandre:

$$f(a+h) - f(a-h) = 2f'(a)h + \frac{f'''(\xi_1)h^3}{6} + \frac{f'''(\xi_2)}{6}h^3$$

flytte over, dele med 2h

$$\underbrace{f'(a) - \frac{f(a+h) - f(a-h)}{2h}}_{\text{symmetrisk newton}} = \underbrace{-\frac{f'''(\xi_1)h^2}{12} - \frac{f'''(\xi_2)}{12}h^2}_{\text{matematisk feil : symmetrisk newton.}}$$

(b): Vi skriver som før $\overline{f(a+h)} = f(a+h)(1+\varepsilon_1)$

$\overline{f(a-h)} = f(a-h)(1+\varepsilon_2)$

(c): $M_1 = \max|f'''(\xi)| \quad M_2 = \max|f(x)|$

10.4.6

newtons metode / sekantmetoden for $f(x) = x^2 - 2$ $\qquad f'(x) = 2x$

sett $e_n = x_n - \sqrt{2}$

Newtons metode: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{2x_n^2 - x_n^2 + 2}{2x_n}$

$$= \frac{x_n^2 + 2}{2x_n}$$

trekk fra $\sqrt{2}$ på begge sider

$$x_{n+1} - \sqrt{2} = \frac{x_n^2 + 2}{2x_n} - \sqrt{2}$$

$$e_{n+1} = \frac{x_n^2 + 2 - 2\sqrt{2}x_n}{2x_n} = \frac{x_n^2 - 2\sqrt{2}x_n + 2}{2x_n} = \frac{(x_n - \sqrt{2})^2}{2x_n} = \underline{\underline{\frac{e_n^2}{2x_n}}}$$

b) sekantmetoden: $x_n = x_{n-1} - \frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})} f(x_{n-1})$

$$= x_{n-1} - \frac{x_{n-1} - x_{n-2}}{x_{n-1}^2 - 2 - ((x_{n-2})^2 - 2)} (x_{n-1}^2 - 2)$$

$(x_{n-1} + x_{n-2})(x_{n-1} - x_{n-2})$

$$= x_{n-1} - \frac{x_{n-1} - x_{n-2}}{x_{n-1}^2 - x_{n-2}^2} (x_{n-1}^2 - 2) = x_{n-1} - \frac{x_{n-1}^2 - 2}{x_{n-1} + x_{n-2}}$$

trekk fra $\sqrt{2}$ på begge sider ($e_n = x_n - \sqrt{2}$, $e_{n-1} = x_{n-1} - \sqrt{2}$)

$$e_n = e_{n-1} - \frac{x_{n-1}^2 - 2}{x_{n-1} + x_{n-2}} = e_{n-1} - \frac{\overbrace{(x_{n-1} - \sqrt{2})}^{e_{n-1}}(x_{n-1} + \sqrt{2})}{x_{n-1} + x_{n-2}}$$

$$= e_{n-1} - e_{n-1}\frac{x_{n-1} + \sqrt{2}}{x_{n-1} + x_{n-2}} = e_{n-1}\left(1 - \frac{x_{n-1} + \sqrt{2}}{x_{n-1} + x_{n-2}}\right)$$

$$= e_{n-1}\frac{x_{n-1} + x_{n-2} - x_{n-1} - \sqrt{2}}{x_{n-1} + x_{n-2}} = e_{n-1}\frac{x_{n-2} - \sqrt{2}}{x_{n-1} + x_{n-2}} = \frac{e_{n-1}e_{n-2}}{x_{n-1} + x_{n-2}}$$

11.1.2

$f(x) = \cos x$, $f''(x) = -\cos x$

$\max_{x\in[a,a+h]} |f''(x)| = 1$

(a) $f'(a) \approx \frac{f(a+h) - f(a)}{h}$

Vi vet: matematisk feil $\leq \frac{h}{2} \max_{x\in[a,a+h]} |f''(x)| = \frac{h}{2}$

derfor er det siste svaralternativet riktig.
(Vi så her bort fra avrundingsfeil)

(b) Vi har lært at total feil (med avrundingsfeil) er

$$\leq \underbrace{\frac{h}{2} \max_{x\in[a,a+h]} |f''(x)|}_{\text{matematisk feil}} + \underbrace{\frac{2\varepsilon^*}{h} \max_{x\in[a,a+h]} |f(x)|}_{\text{avrundingsfeil}}$$

derfor er det siste svaralternativet riktig her også.

11.1.5

Lemma 11.15: optimal steglengde $h^* \approx 2\sqrt{\frac{\epsilon^* |f(a)|}{|f''(a)|}}$ $\quad f(x) = e^x$, $f''(x) = e^x$

$= 2\sqrt{\epsilon^*} = 2\sqrt{7 \times 10^{-17}}$

$\approx 1.6733 \times 10^{-8}$

Dette stemmer bra overens med at feilen i koden ble minst for $p = 8$, dvs $h = 10^{-8}$.

11.3.2

som forrige oppgave (dvs. $f(x) = e^x$), men bruker i stedet symmetrisk newton. $(a=1)$

$f(x) = e^x$

ny formel for optimal steglengde (formel 11.27)

$f'''(x) = e^x$

$$h^* \approx \frac{\sqrt[3]{3\varepsilon^* |f(a)|}}{\sqrt[3]{|f'''(a)|}} = \sqrt[3]{3\varepsilon^*} = \sqrt[3]{3 \cdot 7 \times 10^{-17}} \approx \underline{\underline{5.94 \cdot 10^{-6}}}$$

symmetrisk Newton:

$$\frac{f(\overset{a}{1}+h) - f(\overset{a}{1}-h)}{2h} = \frac{e^{1+h} - e^{1-h}}{2h}$$

12.2.3

Vi skal tilnærme $\int_0^{\pi/2} \frac{\sin x}{1+x^2}\,dx$ med midtpunktsmetoden, 6 delintervaller.

intervallene blir: $[0, \frac{\pi}{12}], [\frac{\pi}{12}, \frac{2\pi}{12}], [\frac{2\pi}{12}, \frac{3\pi}{12}], [\frac{3\pi}{12}, \frac{4\pi}{12}], [\frac{4\pi}{12}, \frac{5\pi}{12}], [\frac{5\pi}{12}, \frac{6\pi}{12}]$

midtpunkter ($\frac{\pi}{12} = \frac{2\pi}{24}$) $\quad \frac{\pi}{24} \quad \frac{3\pi}{24} \quad \frac{5\pi}{24} \quad \frac{7\pi}{24} \quad \frac{9\pi}{24} \quad \frac{11\pi}{24}$

midtpunktsmetoden: $h \sum_{i=1}^{n} f(x_{i-\frac{1}{2}})$ $\qquad (h = \frac{\pi}{12})$

$$= \frac{\pi}{12}\left(f\left(\frac{\pi}{24}\right) + f\left(\frac{3\pi}{24}\right) + f\left(\frac{5\pi}{24}\right) + f\left(\frac{7\pi}{24}\right) + f\left(\frac{9\pi}{24}\right) + f\left(\frac{11\pi}{24}\right)\right)$$

$$= \frac{\pi}{12}\left(\frac{\sin\frac{\pi}{24}}{1+(\frac{\pi}{24})^2} + \frac{\sin\frac{3\pi}{24}}{1+(\frac{3\pi}{24})^2} + \frac{\sin\frac{5\pi}{24}}{1+(\frac{5\pi}{24})^2} + \cdots \right) \approx \underline{\underline{0.530624}}$$

12.2.4 $f(x) = e^x$.

Skal altså regne ut $\int_0^1 e^x \, dx$ med midtpunktsmetoden.

(dette skal bli $e - 1 \approx 1,71828...$)