Pascals trekant og binomialteoremet.

Vi vet at $(a+b)^2 = a^2 + 2ab + b^2$

Hva er $(a+b)^n$, $n > 2$, $n \in \mathbb{N}$

$n=3$

$$(a+b)^3 = (a+b)^2(a+b) = (a^2+2ab+b^2)(a+b)$$

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$= a^3 + (2+1)a^2b + (1+2)ab^2 + b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$n=4$

$$(a+b)^4 = (a+b)^3(a+b)$$

$$= (a^3 + 3a^2b + 3ab^2 + b^3)(a+b)$$

$$= a^4 + 3a^3b + 3a^2b^2 + ab^3$$
$$+ a^3b + 3a^2b^2 + 3ab^3 + b^4$$

$$= a^4 + (3+1)a^3b + (3+3)a^2b^2 + (1+3)ab^3 + b^4$$

Vi må ha

$$(a+b)^n = a^n + z_1 a^{n-1} b + \cdots + z_i a^{n-i} b^i + \cdots + z_{n-1} a b^{n-1} + b^n$$

Hva er z_i'ene?

Pascals trekant

n											
0						1					
1					1		1				
2				1		2		1			
3			1		3		3		1		
4		1		4		6		4		1	
5	1		5		10		10		5		1

⋮

Vil ha formel for z_i'ene, altså tallene i Pascals trekant.

Påstand.
$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

der $\binom{n}{i} = \frac{n!}{(n-i)!\,i!}$, $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$

Altså $z_i = \binom{n}{i}$

Plassert i trekant struktur:

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\vdots$$
$$\binom{n}{i} \quad \binom{n}{i+1}$$
$$\binom{n+1}{i+1}$$

Lemma 1.4.4

$$\binom{n+1}{i} = \binom{n}{i-1} + \binom{n}{i}, \quad i, n \in \mathbb{N}, \; n \geq i.$$

Bevis: Lurt å begynne på høyre side

$$\binom{n}{i-1} + \binom{n}{i} = \frac{n!}{(n-i+1)!\,(i-1)!} + \frac{n!}{(n-i)!\,i!}$$

$i! = (i-1)! \cdot i$

$$= \frac{n!}{(n-i)!\,(i-1)!}\left(\frac{1}{n-i+1} + \frac{1}{i}\right)$$

$$= \frac{n!}{(n-i)!\,(i-1)!}\left(\frac{i + n-i+1}{(n-i+1)\,i}\right) \qquad (n+1-i)!$$

$$= \frac{n!}{(n-i)!\,(i-1)!} \cdot \frac{n+1}{(n-i+1)\cdot i} = \frac{(n+1)!}{(n-i+1)!\,i!}$$

$$= \binom{n+1}{i}$$