Pascals trekant og binomialteoremet.

Vi vet at $(a+b)^2 = a^2 + 2ab + b^2$

Hva er $(a+b)^3$?

$$(a+b)^3 = (a+b)^2(a+b) = (a^2 + 2ab + b^2)(a+b)$$

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$= a^3 + (2+1)a^2b + (1+2)ab^2 + b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = (a+b)^3(a+b) = (a^3 + 3a^2b + 3ab^2 + b^3)(a+b)$$

$$= a^4 + 3a^3b + 3a^2b^2 + ab^3 + a^3b + 3a^2b^2 + 3ab^3 + b^4$$

$$= a^4 + (3+1)a^3b + (3+3)a^2b^2 + (1+3)ab^3 + b^4$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Fins det en formel for $(a+b)^n$?

Vi må ha

$$(a+b)^n = a^n + z_1 a^{n-1} b + z_2 a^{n-2} b^2 + \dots + z_{n-1} a b^{n-1} + b^n$$

Hva er z_i'ene ?

$n=1$, $a+b$

$n=2$, $(a+b)^2 = a^2 + 2ab + b^2$

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

Pascals trekant.

$$(a+b)^5 = a^5 + 5a^4 b + 10 a^3 b^2 + 10 a^2 b^3 + 5 a b^4 + b^5$$

Binomial formelen:

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

der $\binom{n}{i} = \frac{n!}{(n-i)!\,i!}$, $n! = 1\cdot 2\cdot 3\cdot 4\cdots n$.

$\binom{n}{i}$ - binomial koeffisient.

Dette betyr at det er binomial koeff. vi finner i Pascals trekant.

$$\overset{1}{\binom{0}{0}}$$

$$1\,\binom{1}{0} \qquad \binom{1}{1}\,1$$

$$1\,\binom{2}{0} \qquad \binom{2}{1}^{2} \qquad \binom{2}{2}^{1}$$

$$\binom{3}{0}^{1} \qquad \binom{3}{1}^{3} \qquad \binom{3}{2}^{3} \qquad \binom{3}{3}^{1}$$

$$\binom{n}{i} = \frac{n!}{(n-i)!\cdot i!} , \qquad \binom{1}{0} = \frac{1!}{1!\cdot 0!} = \frac{1}{1\cdot 1} = 1$$

$$\binom{2}{1} = \frac{2!}{1!\cdot 1!} = \frac{1\cdot 2}{1\cdot 1} = 2$$

$$\binom{3}{2} = \frac{3!}{1!\cdot 2!} = \frac{1\cdot 2\cdot 3}{1\cdot(1\cdot 2)} = 3$$

Lemma 1.4.4. For alle naturlige tall n og i med $0 \le i \le n$ er

$$\binom{n+1}{i} = \binom{n}{i-1} + \binom{n}{i}$$

- kjerneoperasjonen i Pascals trekant.

$$\binom{n}{i-1} \quad \binom{n}{i} \quad \binom{n+1}{i}$$

Bevis: Husk $\binom{n}{i} = \frac{n!}{(n-i)! \cdot i!}$

Vi begynner med det kompliserte og forenkler.

$$\binom{n}{i-1} + \binom{n}{i} = \frac{n!}{(n-i+1)!\,(i-1)!} + \frac{n!}{(n-i)! \cdot i!}$$

$$(n-(i-1))! = (n-i+1)!$$

$$= \frac{n!}{(i-1)!\,(n-i)!}\left(\frac{1}{(n-i+1)\cdot 1} + \frac{1}{1 \cdot i}\right)$$

$$i! = 1 \cdot 2 \cdot 3 \cdots (i-1) \cdot i = (i-1)! \cdot i$$

$$(n-i+1)! = (n-i)!\,(n-i+1)$$

$$= \frac{n!}{(n-i)!\,(i-1)!}\left(\frac{i}{(n-i+1)\,i} + \frac{(n-i+1)}{(n-i+1)\,i}\right)$$

$$= \frac{n!}{(n-i)!\,(i-1)!}\left(\frac{n+1}{(n-i+1)\,i}\right) = \frac{n!\,(n+1)}{(n-i)!\,(n-i+1)\cdot(i-1)! \cdot i}$$

$$= \frac{(n+1)!}{(n-i+1)!\,i!} = \binom{n+1}{i}$$