Compulsory project 1 in MAT-INF3100 Linear optimization: [1]

- **NOTE: general information:** You should give/send your project by **February 13** to

  Torkel Haufmann [torkelah@math.uio.no]

  either (i) by email in a single pdf-file denoted “username.pdf” (your username!) or (ii) give a paper print to Torkel. You should also read the general information about compulsory projects at the course web page.

Problem 1.

Let $n \geq 1$ and $r_1, r_2, \ldots, r_n > 0$. Consider the LP problem

$$\begin{array}{ll} \max & \sum_{j=1}^n r_j x_j \\ \text{subject to} & \\ & \sum_{j=1}^n x_j \leq 1 \\ & x_1, x_2, \ldots, x_n \geq 0. \end{array} \tag{1}$$

with variables $x_1, x_2, \ldots, x_n$.

a) Let $n = 2$. Draw the feasible set F (i.e., the set of points (x_1, x_2) satisfying the constraints in (1)) in the plane, and solve problem (1) *geometrically* for each value of $r_1, r_2 > 0$.

b) Again let $n = 2$ in (1). Let $\eta(r_1, r_2)$ denote the optimal value in (1). It depends on r_1, r_2 and therefore defines a function $\eta : \mathbb{R}^2 \rightarrow \mathbb{R}$. Determine this function η. Is it differentiable?

c) Consider (1) but for general n (with $r_1, r_2, \ldots, r_n > 0$). Explain why the problem has an optimal solution, and find one. Next, find *all* optimal solutions in (1); the answer here depends on the parameters $r_1, r_2, \ldots, r_n$. Is there an optimal solution x with $\sum_j x_j < 1$?

Problem 2.

a) Use the simplex method to solve the LP problem (by hand)

$$\begin{array}{lrcrcrcl} \max & x_1 & + & 2x_2 & + & x_3 & & \\ \text{subj. to} & & & & & & & \\ & -2x_1 & + & x_2 & & & \leq & 2 \\ & -x_1 & + & 2x_2 & & & \leq & 7 \\ & x_1 & & & + & x_3 & \leq & 3 \\ & & & & & x_1, x_2, x_3 \geq 0. & & \end{array} \tag{2}$$

[1] Geir Dahl, MI/UiO, `geird@math.uio.no`

b) A student wants to buy food for dinner on a rather limited budget. The price of one "unit" of fish, chicken, beef, and apples are, respectively, 35, 20, 60 and 5 (in NOK). Her budget is 50 NOK, and she decides[2] to make her dinner choice by solving the LP problem

$$\begin{array}{lcccccccccc} \text{max} & v_1x_1 & + & v_2x_2 & + & v_3x_3 & + & v_4x_4 & & \\ \text{subj. to} & & & & & & & & & \\ & 35x_1 & + & 20x_2 & + & 60x_3 & + & 5x_4 & \leq & 50 \\ & & & & x_1, x_2, x_3, x_4 \geq 0. & & & & & \end{array} \tag{3}$$

Explain, briefly, why this model makes (some) sense by commenting on the assumptions, limitations, interpretation of the v_j's.

c) Consider the LP problem

$$\begin{array}{ll} \text{max} & \sum_{j=1}^n v_j x_j \\ \text{subj. to} & \\ & \sum_{j=1}^n a_j x_j \leq a_0 \\ & \sum_{j=1}^n b_j x_j \leq b_0 \\ & 0 \leq x_j \leq p_j \quad (j \leq n). \end{array} \tag{4}$$

where the x_j's are the variables and the v_j's, a_j's etc. are nonnegative parameters.

Write an OPL-CPLEX program for the LP problem (4). Write both a mod-file and a dat-file. Set up two test problems (your choice!) and run your program on these test cases. Report the solutions.

Finally, use your program to solve a variation of the student's dinner problem (3) where we add one constraint. Use $v_1 = 4$, $v_2 = 2$, $v_3 = 5$, $v_4 = 1$, and add the constraint

$$7x_1 + 5x_2 + 5x_3 + 10x_4 \leq \alpha$$

where $\alpha > 0$ is a parameter. You may interprete this as a weight constraint (the weight of fish is 700 grams pr unit etc.) and she does not want the total weight to be more that α. Now, solve this problems for a couple of "interesting" values of α. (You have some freedom in the choice of data in this exercise).

Good luck!

[2]being a student of optimization, of course!