

Assignment 1 for MAT-INF4160, 2013

Bezier curves

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To be completed by Tuesday 24 September. Solutions can be handed to me in the lecture or sent electronically to `michael.f@ifi.uio.no`.

1. Show that the Bernstein polynomial $B_i^n(x)$ has only one maximum in $[0, 1]$, namely at $x = i/n$.
2. The Bernstein approximation to a function $f : [0, 1] \rightarrow \mathbb{R}$ of order n is the polynomial $g : [0, 1] \rightarrow \mathbb{R}$ defined by

$$g(x) = \sum_{i=0}^n f(i/n) B_i^n(x).$$

Show that if f is a polynomial of degree $m \leq n$ then g has degree m .

3. Show that

$$\Delta^i \mathbf{c}_0 = \sum_{k=0}^i (-1)^{i-k} \binom{i}{k} \mathbf{c}_k.$$

4. Show that

$$n(n-1) \cdots (n-k)x^{k+1} = \sum_{i=0}^n i(i-1) \cdots (i-k) B_i^n(x).$$