Assignment 2 for MAT-INF4160, 2013

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To be completed by Tuesday 22 October. Solutions can be handed to me in the lecture or sent electronically to `michaelf@ifi.uio.no`.

1. Let $\mathbf{p}(t) = \sum_{i=0}^n \mathbf{c}_i B_i^n(t)$, $\mathbf{c}_i \in \mathbb{R}^d$, be a Bezier curve in $\mathbb{R}^d$. Show that its length,

$$L = \int_0^1 \|\mathbf{p}'(t)\|\, dt,$$

is less than or equal to the length of its control polygon.

2. Consider the two tensor-product Bezier surfaces,

$$\mathbf{p}(s,t) = \sum_{i=0}^m \sum_{j=0}^n \mathbf{c}_{ij} B_i^m(s) B_j^n(t), \qquad (s,t) \in [0,1] \times [0,1],$$

$$\mathbf{q}(s,t) = \sum_{i=0}^m \sum_{j=0}^n \mathbf{d}_{ij} B_i^m(s-1) B_j^n(t), \qquad (s,t) \in [1,2] \times [0,1],$$

with control points $\mathbf{c}_{ij}, \mathbf{d}_{ij} \in \mathbb{R}^3$. What are the conditions on the control points that ensure that $\mathbf{p}$ and $\mathbf{q}$ join with C^1 continuity on the common edge $s = 1$, $0 \le t \le 1$?

3. B-splines: find the coefficients $c_i \in \mathbb{R}$ such that

$$t^2 = \sum_{i=1}^n c_i N_i^3(t), \qquad t_4 \le t \le t_{n+1},$$

over some knot vector $t_1, t_2, \ldots, t_{n+4}$,

4. (a) Write a computer program which, given a, b, with $a < b$, and $\mathbf{c}_0, \ldots, \mathbf{c}_n \in \mathbb{R}^2$, plots the Bezier curve $\mathbf{p}(t) = \sum_{i=0}^n \mathbf{c}_i B_i(u)$, where $u = (t-a)/(b-a)$.

(b) Use the program to plot the composite curve $\mathbf{s} : [0,2] \to \mathbb{R}^2$, with pieces

$$\mathbf{p}(t) = \sum_{i=0}^{2} \mathbf{c}_i B_i^2(t), \qquad 0 \leq t < 1,$$

$$\mathbf{q}(t) = \sum_{i=0}^{2} \mathbf{d}_i B_i^2(t-1), \qquad 1 \leq t < 2,$$

where $\mathbf{c}_0 = (-1,1)$, $\mathbf{c}_1 = (-1,0)$, $\mathbf{c}_2 = (0,0)$, and $\mathbf{d}_0 = (0,0)$, $\mathbf{d}_1 = (1,0)$, $\mathbf{d}_2 = (2,1)$. What is the order of continuity of $\mathbf{s}$ at $(0,0)$?

(c) The *curvature* of a parametric curve $\mathbf{r}$ in $\mathbb{R}^2$ is

$$\kappa(\mathbf{r}(t)) = \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\|\mathbf{r}'(t)\|^3},$$

where $(a_1, a_2) \times (b_1, b_2) := a_1 b_2 - a_2 b_1$. What are the curvatures of $\mathbf{p}$ and $\mathbf{q}$ at $(0,0)$?