Assignment 3 for MAT-INF4160, 2013

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To be completed by Tuesday 19 November. Solutions can be handed to me in the lecture or sent electronically to `michaelf@ifi.uio.no`.

1. Hermite interpolation: find the Bezier control points $\mathbf{c}_i$ in the quintic representation

$$\mathbf{p}(t) = \sum_{j=0}^{5} \mathbf{c}_j B_j^5(u),$$

where $u = (t-a)/(b-a)$, such that $\mathbf{p}^{(i)}(a) = \mathbf{f}^{(i)}(a)$ and $\mathbf{p}^{(i)}(b) = \mathbf{f}^{(i)}(b)$, $i = 0, 1, 2$.

2. Implement C^2 cubic spline curve interpolation. Given points $\mathbf{x}_i \in \mathbb{R}^2$, $0 \leq i \leq n$, choose some parameter values $t_0 < t_1 < \cdots < t_n$ and find the unique C^2 cubic spline curve $\mathbf{s} : [t_0, t_n] \to \mathbb{R}^2$ such that $\mathbf{s}(t_i) = \mathbf{x}_i$, $0 \leq i \leq n$, and $\mathbf{s}'(t_0) = \mathbf{s}'(t_n) = \mathbf{0}$. Make plots of the curve $\mathbf{s}$ for the three choices of parameterization

$$t_{i+1} - t_i = \|\mathbf{x}_{i+1} - \mathbf{x}_i\|^{\mu},$$

with $\mu = 0, 1/2, 1$, and for the following two data sets. The first data set is $\mathbf{x}_i = (x_i, y_i)$, where

$$(x_0, \ldots, x_9) = (261, 261, 283, 287, 280, 281, 302, 319, 335, 277),$$
$$(y_0, \ldots, y_9) = (703, 738, 718, 723, 735, 736, 731, 748, 737, 682).$$

The second data set is $\mathbf{x}_i = (\cos(s_i), \sin(s_i))$, where

$$(s_0, s_1, \ldots, s_{12}) = $$
$$(0, 0.24, 0.39, 0.78, 1.03, 1.18, 1.56, 1.81, 1.96, 2.34, 2.59, 2.74, 3.12).$$

What effects of the three parameterizations do you observe?