

Oblig 3 - exercise 4.5: Hint

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4.5 Prove Lemma 4.2 in the general case where τ and t are not $p+1$ -regular. Hint: Augment both τ and t by inserting $p + 1$ identical knots at the beginning and end.

Show that any $f \in \mathbb{S}_\tau$ also lies in \mathbb{S}_t

Extended Hint:

Insert $p+1$ knots into τ at a position $a < t_1$ and $p+1$ knots at a position $b > t_{m+p+1}$. Denote this knot vector τ' . We also insert the same knots into t to form t' .

Adding $p+1$ knots at both ends extends the corresponding splinespaces by adding $p+1$ B-splines at both ends. It is easy to see that $\mathbb{S}_\tau \subseteq \mathbb{S}_{\tau'}$ since a $f \in \mathbb{S}_\tau$ can be represented in $\mathbb{S}_{\tau'}$ by adding $p+1$ zero-coefficients in both ends. We also know that $\mathbb{S}_{\tau'} \subseteq \mathbb{S}_{t'}$ by Lemma 4.2 (since τ' and t' are $p+1$ -regular) and hence $\mathbb{S}_\tau \subseteq \mathbb{S}_{\tau'} \subseteq \mathbb{S}_{t'}$.

Take $f \in \mathbb{S}_\tau$ and show that $f \in \mathbb{S}_t$.