MAT0011 onsdag 10. august

Kap. 4 Rasjonale uttrykk

$\frac{P(x)}{Q(x)}$, der $P(x)$ og $Q(x)$ er polynomer

1) Hvis graden til $P(x) \geq$ graden til $Q(x)$: polynomdivisjon

2) Hvis graden til $P(x) <$ — " —— : delbrøkoppspalting, dvs. dele opp uttrykket "i mindre biter"

Tilfelle 1: $\frac{P(x)}{(x-r_1)(x-r_2)\cdots(x-r_n)}$ der $r_1, r_2, \ldots, r_n$ er de ulike røttene til likningen $Q(x)=0$ (graden til $Q(x) = n$)

$$= \frac{A_1}{(x-r_1)} + \frac{A_2}{(x-r_2)} + \ldots + \frac{A_n}{(x-r_n)}$$

eks. $\frac{4x+3}{x^2+2x-3}$

1) faktorisere nevneren: $x^2+2x-3=0$

$$x = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm 4}{2} = \begin{cases} 1 \\ -3 \end{cases}$$

$$(x-1)(x+3) = 0$$

$$\frac{4x+3}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$$

A, B ukjente

$$4x+3 \underset{\substack{\uparrow \\ \text{skal gjelde for alle } x}}{=} \underbrace{A(x+3)} + \underbrace{B(x-1)}$$

Alt 1: Velge lure verdier for x:

$x = 1$: $7 = A \cdot 4$

$A = \frac{7}{4}$

$x = -3$: $-9 = B \cdot (-4)$

$B = \frac{9}{4}$

Alt. 2:

$4x+3 = Ax + 3A + Bx - B$

$4x + 3 = (A+B)x + 3A - B$

$$\begin{cases} 4 = A + B & \text{(x-leddene)} \\ 3 = 3A - B & \text{(konstantene)} \end{cases}$$

$7 = 4A$

$A = \frac{7}{4}$

$4 = \frac{7}{4} + B$

$\frac{9}{4} = B$

$$\frac{4x+3}{x^2+2x-3} = \frac{\frac{7}{4}}{x-1} + \frac{\frac{9}{4}}{x+3}$$

Tilfelle 2:

$$\frac{S(x)}{(x-r)^n} = \frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + \dots + \frac{A_n}{(x-r)^n}$$

kan finne $A_1, \dots, A_n$

eks. $$\frac{x}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

kan gjøre som i forrige eksempel (alt. 1 & 2)

Triks:

$$\frac{x}{(x-1)^2} = \frac{\overbrace{x-1+1}^{=0}}{(x-1)^2} = \frac{x-1}{(x-1)^2} + \frac{1}{(x-1)^2} = \underline{\underline{\frac{1}{x-1} + \frac{1}{(x-1)^2}}}$$

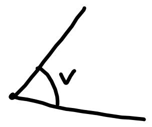
$(A = B = 1)$

Flere tilfeller: MAT1100

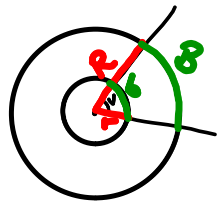
Kap.5

Trigonometri

trekantmåling



naturlig vinkelmål:
radianer



$$\frac{b}{2\pi r} = \frac{B}{2\pi R}$$

$$\frac{b}{r} = \frac{B}{R}$$

$$\text{radian} = \frac{\text{buelengde}}{\text{radius}}$$

Hel sirkel: $360° = \frac{2\pi r}{r} = 2\pi$ (radianer)

Likhet mellom vinkelmål: $\boxed{360° = 2\pi}$

$$\begin{cases} 180° = \pi \\ 90° = \frac{\pi}{2} \\ 45° = \frac{\pi}{4} \end{cases}$$

$60° = \frac{\pi}{3}$

$30° = \frac{\pi}{6}$

$75° = 60° + 15°$
$= \frac{\pi}{3} + \frac{\pi}{12}$
$= \frac{5\pi}{12}$

1 rad ?

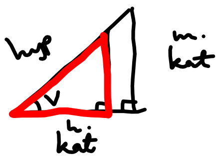
$1 \text{ rad} = \frac{\text{buelengden}}{\text{radius}}$

dvs. $b = r$

$\frac{360°}{2\pi} \approx 57{,}6°$

pause til 10^{28}

Trigonometri:



$$\boxed{\begin{aligned} \sin v &= \frac{\text{motstående katet}}{\text{hypotenus}} \\ \cos v &= \frac{\text{hosliggende katet}}{\text{hypotenus}} \end{aligned}}$$

$\tan v = \dfrac{\sin v}{\cos v}$

$\cot v = \dfrac{\cos v}{\sin v}$ kotangens

$\sec v = \dfrac{1}{\cos v}$ sekans

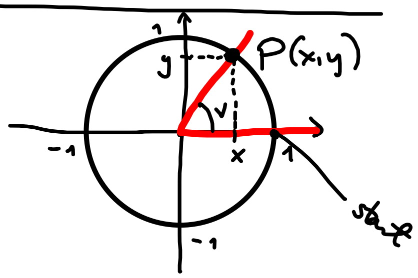
$\csc v = \dfrac{1}{\sin v}$ kosekans

($\dfrac{1}{\cos v} \neq \cos^{-1} v$ på kalkulator

$\cos^{-1} v = a\cos v = \arccos v$)

Enhetssirkelen:

sirkelen med sentrum i origo og radius lik 1



Vinkel v legges inn med toppunkt i origo og dets ene vinkelbeint langs den pos. førsteaksen.

Det andre vinkelbeinet vil skjære enhetssirkelen i et punkt P.

$\cos v = \dfrac{x}{1}$

$\sin v = \dfrac{y}{1}$

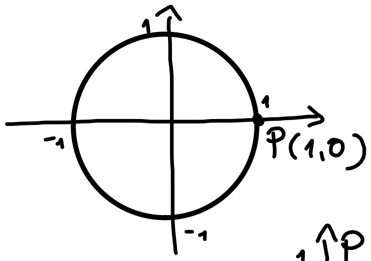
$$\boxed{\begin{aligned} \cos v &= x \\ \sin v &= y \end{aligned}}$$ Definisjon

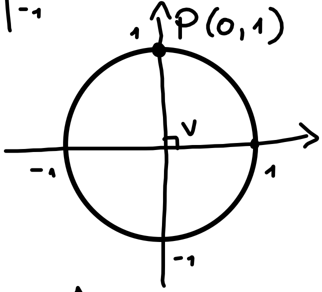
$v \in \mathbb{R}$

$\underline{v = 0}$:

$\cos 0 = 1$

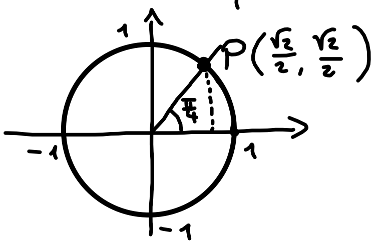
$\sin 0 = 0$





$\underline{v = \frac{\pi}{2}}$: $\cos\frac{\pi}{2} = 0$

$\sin\frac{\pi}{2} = 1$



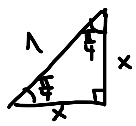
$\underline{v = \frac{\pi}{4}}$: $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$x^2 + x^2 = 1$

$2x^2 = 1$

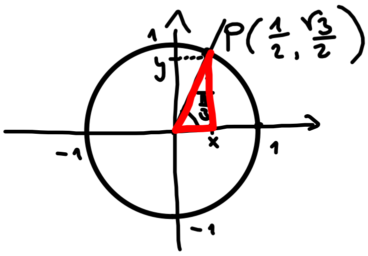
$x^2 = \frac{1}{2}$



Likebeint trekant

$x = \pm\sqrt{\frac{1}{2}}$

$x = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \underline{\frac{\sqrt{2}}{2}}$



$\underline{v = \frac{\pi}{3}}$: $\cos\frac{\pi}{3} = \frac{1}{2}$

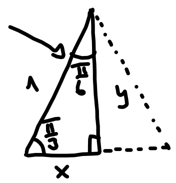
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\left(\frac{1}{2}\right)^2 + y^2 = 1$

$\frac{1}{4} + y^2 = 1$

$y^2 = \frac{3}{4}$

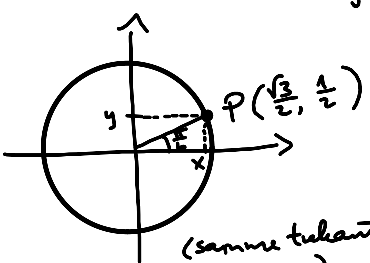
$y = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \underline{\frac{\sqrt{3}}{2}}$



$x = \frac{1}{2}$

halvt likesidet

$\underline{v = \frac{\pi}{6}}$:



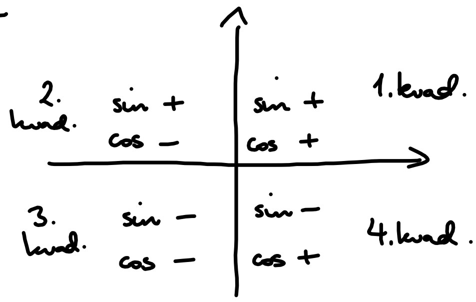
(samme trekant som over)

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

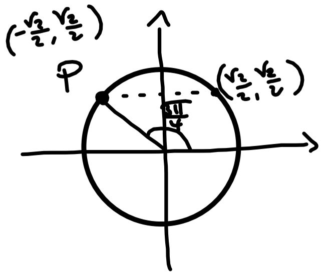
$\sin\frac{\pi}{6} = \frac{1}{2}$

Tabell:

v	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin v$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0



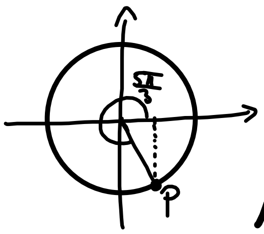
$v = \frac{3\pi}{4}$



$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

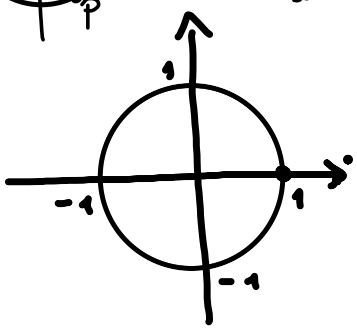
$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$v = \frac{5\pi}{3}$



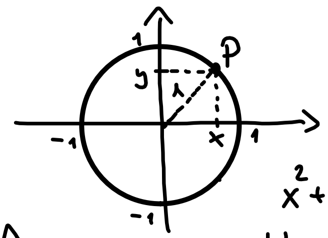
$\cos\frac{5\pi}{3} = \frac{1}{2}$

$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$



pause til 11^{28}

Trigonometriske formler:



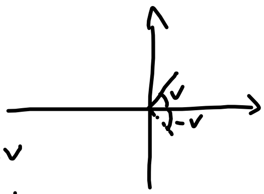
$x^2 + y^2 = 1$

Likningen for enhetssirkelen

$\cos^2 v + \sin^2 v = 1$

$\cos(-v) = \cos v$

$\sin(-v) = -\sin v$



$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\sin(u+v) = \underline{\cos u \sin v + \cos v \sin u}$

$\cos 2u = \cos^2 u - \sin^2 u$

$\sin 2u = 2 \sin u \cos u$

Trigonometriske funksjoner:

$f(x) = \sin x$ $\qquad$ $g(x) = \cos x$

$D_f = \mathbb{R}$ $\qquad$ $D_g = \mathbb{R}$

$V_f = [-1, 1]$ $\qquad$ $V_g = [-1, 1]$

$h(x) = \tan x$

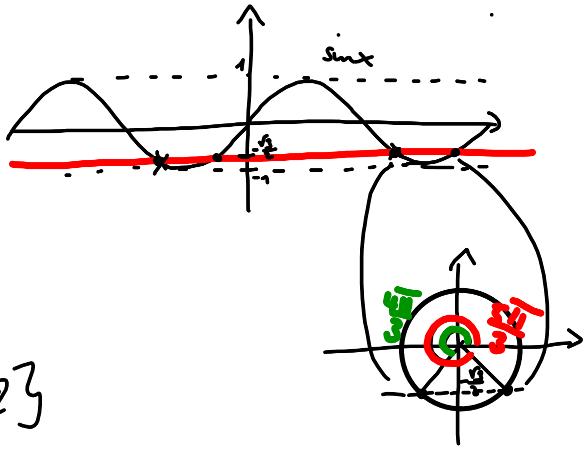
$= \frac{\sin x}{\cos x}$

$D_h = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$

Grafer: se ark

Trigonometriske likninger:

1) Grunnlikninger:
La $a \in \mathbb{R}$.
$\sin x = a$



eks: $\sin x = -\frac{\sqrt{3}}{2}$

$$x = \left\{ \frac{4\pi}{3} + 2\pi k \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{3} + 2\pi k \mid k \in \mathbb{Z} \right\}$$

2) Triks:

$a \sin x + b \cos x = 0$, a, b gitt $\quad | : \cos x$ (antar at $\cos x \neq 0$)

$$\frac{a \sin x}{\cos x} + b = 0$$

$$a \tan x + b = 0$$

$$\tan x = -\frac{b}{a} \quad \text{grunnlikning}$$

Andre triks: husk formlene

Standardform:

Gitt $a, b \in \mathbb{R}$.
Vi kan skrive

$$a\sin x + b\cos x = A\sin(x+\varphi)$$

φ "fi"

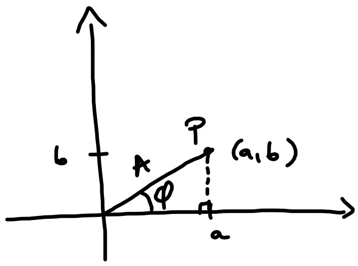
der $A = \sqrt{a^2+b^2}$ og $a = A\cos\varphi$ og $b = A\sin\varphi$.

Hvorfor?

$$A\sin(x+\varphi) = A(\sin x\cos\varphi + \sin\varphi\cos x)$$
$$= \underbrace{A\cos\varphi}_{a}\sin x + \underbrace{A\sin\varphi}_{b}\cos x$$

$$\sqrt{a^2+b^2} = \sqrt{(A\cos\varphi)^2 + (A\sin\varphi)^2} = \sqrt{A^2\cos^2\varphi + A^2\sin^2\varphi}$$
$$= \sqrt{A^2(\cos^2\varphi + \sin^2\varphi)} = \sqrt{A^2} = A$$

Geometrisk:



$A = \sqrt{a^2+b^2}$

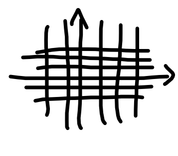
$a = A\cos\varphi$ $\quad \cos\varphi = \frac{a}{A}$

$b = A\sin\varphi$ $\quad \sin\varphi = \frac{b}{A}$

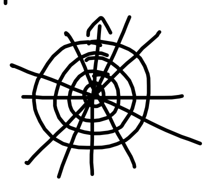
(a, b): kartesiske koordinatene til P

(A, φ): polarkoordinatene til P

kartesisk:



polar:



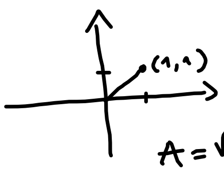
eks.

$$\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

amplituden

faseforskyvning

$a = 1, b = 1$



$A = \sqrt{2}$

$\varphi = \frac{\pi}{4}$

I morgen: grenseverdier!