# Regneregler

Wednesday, November 12, 2014 12:08 PM

**Regneregler**

Hvis $a \equiv b \pmod{n}$ og $c \equiv d \pmod{n}$, så gjelder

1) $a + c \equiv b + d \pmod{n}$
2) $a \cdot c \equiv b \cdot d \pmod{n}$

**Eksempel:** $27 \equiv 3 \pmod{12}$, $19 \equiv 7 \pmod{12}$

teoremet sier at da er $27 + 19 \equiv 7 + 3 \pmod{12}$

dvs at $46 \equiv 10 \pmod{12}$

Hvorfor stemmer dette?

$$27 = 3 + 2 \cdot 12$$
$$19 = 7 + 12$$

$$
\begin{aligned}
27 + 19 &= 3 + 2 \cdot 12 + 7 + 12 \\
&= 3 + 7 + 3 \cdot 12 \\
&\equiv 3 + 7 \pmod{12}
\end{aligned}
$$

generelt tilfelle for punkt 2)

La $a \equiv b \pmod{n}$, $c \equiv d \pmod{n}$

dvs at $a = b + k \cdot n$, $c = d + m \cdot n$ der $k, m$ er heltall

$$
\begin{aligned}
a \cdot c &= (b + k \cdot n)(d + m \cdot n) \\
&= b \cdot d + b \cdot m \cdot n + k \cdot n \cdot d + k \cdot n \cdot m \cdot n \\
&= b \cdot d + (b \cdot m + k \cdot d + k \cdot n \cdot m) \cdot n \\
&\equiv b \cdot d \pmod{n}
\end{aligned}
$$

# Oppg. 46

Wednesday, November 12, 2014 1:23 PM

Finn minste positive $x$ slik at

a) $16 + 23 \equiv x \pmod 7$

$16 \equiv 2 \pmod 7$
$23 \equiv 2 \pmod 7$

$16+23 \equiv 2+2 \pmod 7$
$= 4$

Hvorfor kan vi ikke velge mindre positiv $x$ enn 4?
Siden $0 \le 4 < 7$ så er 4 resten når vi deler $16+23$ på 7, altså
$16+23 = 4 + 5 \cdot 7$.
Det finnes kun ett heltall $r$ i $[0,7)$ slik at $16+23$ er på formen
$$16+23 = r + q \cdot 7$$
der $q$ er et heltall

c) $410 + 408 \equiv x \pmod{401}$

$410 \equiv 9 \pmod{401}$
$408 \equiv 7 \pmod{401}$
$410 + 408 \equiv 9 + 7 \pmod{401}$
$= \underline{\underline{16}}$

d) $16 \cdot 23 \equiv x \pmod 7$

$16 \equiv 2 \pmod 7$
$23 \equiv 2 \pmod 7$

$16 \cdot 23 \equiv 2 \cdot 2 \pmod 7$
$\underline{\underline{x = 4}}$

e) $71 \cdot 23 \equiv x \pmod 5$

$71 \equiv 1 \pmod 5$
$23 \equiv 3 \pmod 5$

$71 \cdot 23 \equiv 1 \cdot 3 \pmod 5$
$\underline{\underline{x = 3}}$

# Oppg. 47-50

Wednesday, November 12, 2014 2:06 PM

47) Hva blir resten når $794 \cdot 31$ delers på 9 ?

$$\begin{aligned} 794 &= 720 + 72 + 2 \\ &= 9 \cdot 80 + 9 \cdot 8 + 2 \\ &\equiv 2 \pmod 9 \end{aligned}$$

$$\begin{aligned} 31 &= 3 \cdot 9 + 4 \\ &\equiv 4 \pmod 9 \end{aligned}$$

$$\begin{aligned} 794 \cdot 31 &\equiv 2 \cdot 4 \pmod 9 \\ &= 8 \end{aligned}$$

$\underline{\underline{r = 8}}$

48) Hva blir resten når $\underbrace{743589}_{a} \cdot \underbrace{301647}_{b}$ deles på 5 ?

$a \equiv 4 \pmod 5$

$b \equiv 2 \pmod 5$

$$\begin{aligned} a \cdot b &\equiv 4 \cdot 2 \pmod 5 \\ &\equiv 3 \pmod 5 \end{aligned}$$

$\underline{\underline{r = 3}}$

**Lemma 13 og 14**

- et helt tall $a$ er et partall hvis og bare hvis $a \equiv 0 \pmod 2$
- et heltall $a$ er et oddetall hvis og bare hvis $a \equiv 1 \pmod 2$

49) Vis at summen av to oddetall er et partall

La $a, b$ være oddetall, dvs $a \equiv b \equiv 1 \pmod 2$

$a + b \equiv 1 + 1 \equiv 0 \pmod 2$

dermed er $a+b$ et partall

50) Vis at summen av tre oddetall er et oddetall

La $a, b, c$ være oddetall, dvs $a \equiv b \equiv c \equiv 1 \pmod 2$

vet at $(a+b) \equiv 0$

$$\begin{aligned} a+b+c &= (a+b) + c \\ &\equiv 0 + 1 \pmod 2 \\ &= 1 \end{aligned}$$

dermed er $a+b+c$ et oddetall

Hvis $a_i \equiv b_i \pmod m$ for $i = 1, \dots, n$

1) $a_1 + a_2 + \dots + a_n \equiv b_1 + b_2 + \dots b_n \pmod m$

2) $a_1 \cdot a_2 \cdot \dots \cdot a_n \equiv b_1 \cdot b_2 \cdot \dots \cdot b_n \pmod m$

# Oppg. 51

Wednesday, November 12, 2014 6:01 PM

**Teorem 17**

Anta at $a \equiv b \pmod{n}$, og at $k$ er et naturlig tall

Da er $a^k \equiv b^k \pmod{n}$

Finn den minste positive $x$ slik at

a) $12^3 \equiv x \pmod{55}$

$12^2 = 144 \equiv 34 \pmod{55}$ fordi $144 = 110 + 34$

$12^2 = 12^2 \cdot 12 \equiv 34 \cdot 12 \pmod{55}$

$= 408 \equiv 23 \pmod{55}$ $408 = 7 \cdot 55 + 23$

$\boxed{12^3 \equiv 23 \pmod{55}}$

b) $23^{27} \equiv x \pmod{55}$

$23^{27} = 23^{3 \cdot 3 \cdot 3} = ((23^3)^3)^3$

- Ser først på $23^3$

$23^2 = 529 \equiv 34 \pmod{55}$ $529 = 9 \cdot 55 + 34$

$23^3 = 23^2 \cdot 23 \equiv 34 \cdot 23 \pmod{55}$

$= 782 \equiv 12 \pmod{55}$ $782 = 12 + 14 \cdot 55$

$\boxed{23^3 \equiv 12 \pmod{55}}$

- $(23^3)^3 \equiv 12^3 \pmod{55}$

$\equiv 23 \pmod{55}$

- $(\underbrace{(23^3)^3}_{\equiv 23})^3 \equiv 23^3 \pmod{55}$

$\equiv 12 \pmod{55}$

$\boxed{\text{dvs } 23^{27} \equiv 12 \pmod{55}}$

c) $12^{81} \equiv x \pmod{55}$

$12^{81} = 12^{3 \cdot 27} = (12^3)^{27} \equiv 23^{27} \pmod{55}$

$\equiv 12 \pmod{55}$

# Oppg. 53

Wednesday, November 12, 2014 6:36 PM

Hva er det siste sifferet i disse tallene

a) $6^{52}$

Vi vil finne resten når vi deler $6^{52}$ på 10.
eller det minste heltallet x slik at

$$6^{52} \equiv x \pmod{10}$$

$$6^2 = 36 \equiv 6 \pmod{10}$$
$$6^4 = (6^2)^2 \equiv 6^2 \equiv 6 \pmod{10}$$
$$6^8 = (6^4)^2 \equiv 6^2 \equiv 6 \pmod{10}$$
$$6^{16} \quad \dots \quad \equiv 6 \pmod{10}$$
$$6^{32} \quad \dots \quad \equiv 6 \pmod{10}$$
$$6^{52} = 6^{32+16+4} = 6^{32} \cdot 6^{16} \cdot 6^4 \equiv (6 \cdot 6) \cdot 6 \equiv 6 \cdot 6 \equiv \underline{\underline{6}} \pmod{10}$$

e) $2^{61} \cdot 3^{62}$

- $a = 2^{61} \cdot 3^{62} = 2^{52} \cdot 3^{52} \cdot 2^9 \cdot 3^{10} = 6^{52} \cdot 2^9 \cdot 3^{10}$
  - ser på $2^9$
    $$2^2 = 4 \equiv 4 \pmod{10}$$
    $$2^4 = (2^2)^2 \equiv 4^2 \pmod{10}$$
    $$\equiv 6 \pmod{10}$$
    $$2^8 = (2^4)^2 \equiv 6^2 \equiv 6 \pmod{10}$$
    $$2^9 = 2^8 \cdot 2 \equiv 6 \cdot 2 \equiv \underline{2} \pmod{10}$$

**Alternativ**

$$2^3 = 8$$
$$2^6 = (2^3)^2 \equiv 4 \pmod{10}$$
$$2^9 = 2^3 \cdot 2^6 \equiv 2 \pmod{10}$$

- $$3^3 = 27 \equiv 7 \pmod{10}$$
  $$3^6 = (3^3)^2 \equiv 7^2 \equiv 9 \pmod{10}$$
  $$3^9 = 3^6 \cdot 3^3 \equiv 9 \cdot 7 \equiv 3 \pmod{10}$$
  $$3^{10} = 3 \cdot 3^9 \equiv 3 \cdot 3 \equiv 9 \pmod{10}$$
- $$a \equiv (6 \cdot 2) \cdot 9 \equiv 2 \cdot 9 \equiv \underline{\underline{8}} \pmod{10}$$

# Oppg. 90 b

Thursday, November 13, 2014 11:52 AM

$$\boxed{\text{Separabel lign.} \quad v' \cdot p(v) = q(x)}$$

$$V' = 4 - k \cdot V$$

$$\frac{v'}{4-k \cdot V} = 1$$

$$\int \frac{v'}{4-k \cdot v}\, dx = \int 1\, dx$$

$$v' = \frac{dv}{dx}$$

$$v'\,dx = dv$$

$$\int \frac{1}{4-k \cdot v}\, dv = \int 1\, dx$$

$$\boxed{\int 1\, dx = x + C}$$

$$\int \frac{1}{4-k \cdot v}\, dv = -\frac{1}{k} \ln|4-kv|$$

$$(\ln|4-kv|)' = \frac{1}{4-k \cdot v} \cdot -k$$

$$-\frac{1}{k} \ln|4-kv| = x + c$$

$$\ln|4-kv| = -k(x+c)$$

$$e^{\ln|4-kv|} = e^{-k(x+c)}$$

$$|4-kv| = e^{-kx} e^{-kc}$$

positiv konstant

$$4-kv = e^{-kx} D$$

konstant i $\mathbb{R}$

$$kv = 4 - e^{kx} \cdot D$$

$$v(x) = \frac{4 - e^{kx} \cdot D}{k}$$

etc...