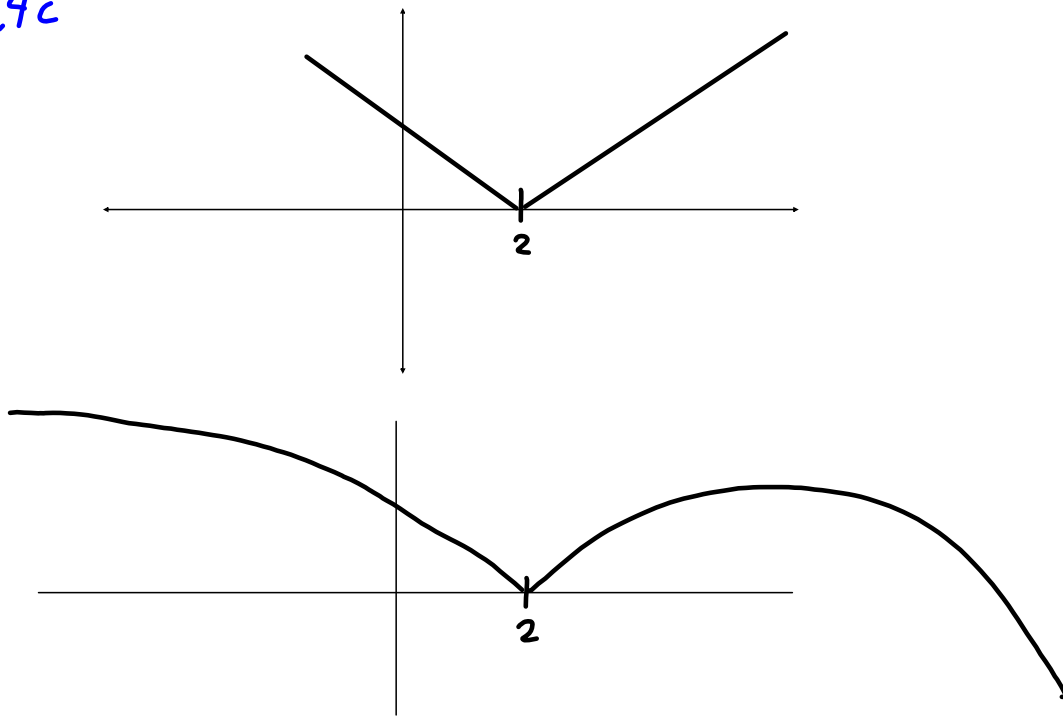


24c



Sep 18-11:04 AM

31) Deriver funksjonene. (Hvilke derivasjonsregler bruker vi?)

a) $f(x) = 3x^4 - 4x + 3$

$$f'(x) = 3 \cdot 4x^3 - 4 + 0 = 12x^3 - 4$$

b) $f(x) = 6x^5 - 3x^4 + 2x^2 - 7$

c) $f(x) = x^{3/2}$

d) $f(x) = x^{-3/2}$

$$f'(x) = -\frac{3}{2} x^{-3/2 - 1} = \underline{\underline{-\frac{3}{2} x^{-5/2}}} = \underline{\underline{\frac{-3}{2 \sqrt{x^5}}}}$$

e) $f(x) = x^{5/3} + 3x^{1/3}$

$$\begin{aligned} (x^r)' &= r x^{r-1} \\ (f+g)' &= f' + g' \end{aligned}$$

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32) Deriver $h(x) = \frac{1}{x} = x^{-1}$

a) bruk $(x^r)' = r x^{r-1}$
 $h'(x) = -1 x^{-1-1} = -\frac{1}{x^2}$

b) bruk $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - g'(x)f(x)}{g(x)^2}$ (kvotientregelen)
 $\left. \begin{matrix} f(x)=1, f'=0 \\ g(x)=x, g'=1 \end{matrix} \right\} h'(x) = \frac{x \cdot 0 - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$

c) deriver $h(x) = \frac{x+1}{x}$ vha kvotientregelen
 $\left. \begin{matrix} f'=1 \\ g'=1 \end{matrix} \right\} h'(x) = \frac{x \cdot 1 - 1 \cdot (x+1)}{x^2} = \underline{\underline{-\frac{1}{x^2}}}$

d) vha svaret i a), b)

$$h(x) = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$$

$$h'(x) = (1)' + \left(\frac{1}{x}\right)' = 0 + -\frac{1}{x^2} = \underline{\underline{-\frac{1}{x^2}}}$$

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33) Husk kjernerregelen

Hvis $f(x) = g(h(x))$, så er
 $f'(x) = g'(h(x)) \cdot h'(x)$

hvis $u = g(x)$, så er
 $f(u)' = f'(u) \cdot u'$

produktregelen

$$(h(x)g(x))' = h'(x)g(x) + h(x)g'(x)$$

a) $x \sqrt{x^2+1} = x(x^2+1)^{1/2}$
 $(x \sqrt{x^2+1})' = x'(x^2+1)^{1/2} + x((x^2+1)^{1/2})'$
 Ser på $\underbrace{(x^2+1)^{1/2}}_u = (u^{1/2})' = \frac{1}{2} u^{-1/2} \cdot u'$
 $= \frac{1}{2} (x^2+1)^{-1/2} \cdot 2x$
 $= \frac{1}{2 \sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$

$$\begin{aligned} &= 1 \cdot \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}} = \frac{x^2+1}{\sqrt{x^2+1}} + \frac{x^2}{\sqrt{x^2+1}} \\ &= \underline{\underline{\frac{2x^2+1}{\sqrt{x^2+1}}}} \end{aligned}$$

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$$b) m(x) = \frac{x-1}{x^2+3} \quad \begin{array}{l} = f(x) \\ = g(x) \end{array}$$

$$\boxed{\begin{array}{l} f'(x) = 1 \\ g'(x) = 2x \end{array}}$$

$$\begin{aligned} m'(x) &= \frac{g \cdot f' - g' \cdot f}{g^2} \\ &= \frac{(x^2+3) \cdot 1 - 2x \cdot (x-1)}{(x^2+3)^2} \\ &= \frac{x^2+3 - 2x^2+2x}{(x^2+3)^2} \\ &= \frac{-x^2+2x+3}{(x^2+3)^2} \end{aligned}$$

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33c) (ikke gjennomgått på kollektium)

$$s(x) = \sqrt{x^3 - x + 1} = (x^3 - x + 1)^{1/2}$$

$$s(x) = f(g(x)) \quad \begin{array}{l} \text{der } f(u) = u^{1/2} \\ u = g(x) = x^3 - x + 1 \end{array}$$

$$\begin{aligned} s'(x) &= f'(u) \cdot g'(x) \\ &= \frac{1}{2} u^{-1/2} \cdot (3x^2 - 1) \\ &= \frac{1}{2} (x^3 - x + 1)^{-1/2} (3x^2 - 1) \end{aligned}$$

$$\boxed{\begin{array}{l} f'(u) = \frac{1}{2} u^{-1/2} \\ g'(x) = 3x^2 - 1 \end{array}}$$

$$= \frac{3x^2 - 1}{2\sqrt{x^3 - x + 1}}$$

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34) stigningstallet til tangenten i $x=3$?

$$f(x) = x^3 + \sqrt{x^2+7}$$

$$f'(x) = (x^3)' + \sqrt{x^2+7}'$$

$$= 3x^2 + \frac{x}{\sqrt{x^2+7}}$$

$$f'(3) = 3 \cdot 3^2 + \frac{3}{\sqrt{3^2+7}}$$

$$= 27 + \frac{3}{\sqrt{16}}$$

$$= \frac{111}{4}$$

$$\sqrt{x^2+7}' = (x^2+7)^{1/2} = u^{1/2}$$

$$(\sqrt{x^2+7})' = (u^{1/2})' \cdot u'(x)$$

$$= \frac{1}{2} u^{-1/2} \cdot 2x$$

$$= \frac{1}{2} (x^2+7)^{-1/2} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+7}}$$

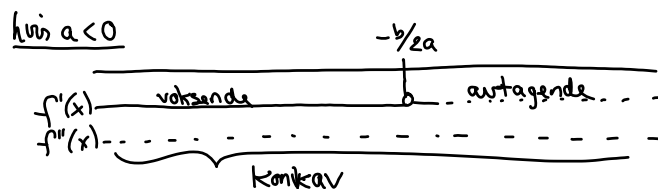
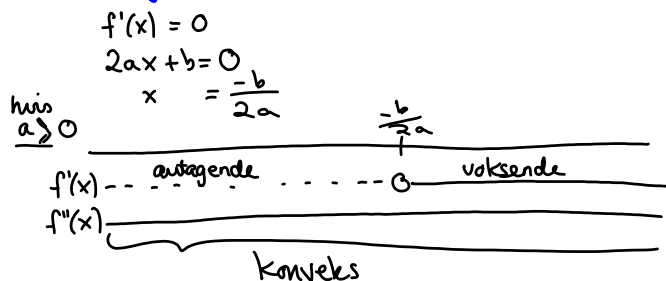
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7

37) $a, b, c, d \in \mathbb{R}$ Find dobbelt-derivert.

b) $f(x) = ax^2 + bx + c$
 $f'(x) = a \cdot 2x + b = 2ax + b$
 $f''(x) = 2a$

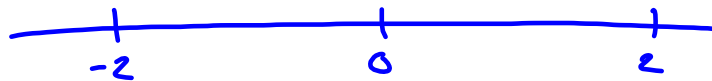
38) Hvor er f konvekst? konkav? voksende? aftagende? Maks/Min?



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39) Finn null-, maksimums-, minimums- og vendepunkter.
 Når er funksjonen konveks, konkav, økende, avtagende?

a) $f(x) = x^3$ på $[-2, 2]$
 $f'(x) = 3x^2$
 $f''(x) = 3 \cdot 2x = 6x$



$f(x) = x^3$
 $f'(x) = 3x^2$
 $f''(x) = 6x$

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39b) $g(x) = \frac{1}{x^2+1} = (x^2+1)^{-1} = u^{-1}$
 $\begin{matrix} u = x^2+1 \\ u' = 2x \end{matrix}$

$g'(x) = -1u^{-2} \cdot u'$
 $= -(x^2+1)^{-2} \cdot 2x$
 $= \frac{-2x}{(x^2+1)^2}$

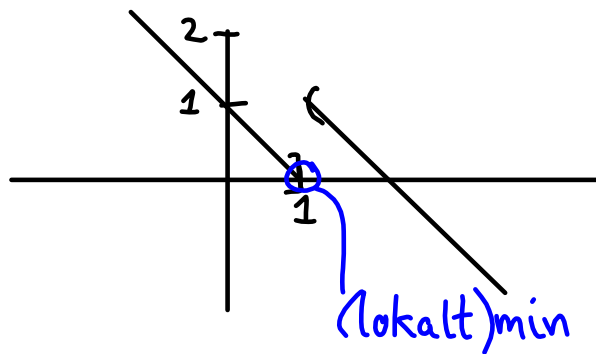
$g''(x) = \frac{h \cdot f' - h' \cdot f}{h^2}$
 $= \frac{(x^2+1)^2(-2) - 4x(x^2+1)(2x)}{(x^2+1)^4}$
 $= \frac{-2(x^2+1)(1-3x^2)}{(x^2+1)^4}$
 $= \frac{-2(1-3x^2)}{(x^2+1)^3}$

$f' = -2$
 $h' = 2(x^2+1) \cdot 2x = 4x(x^2+1)$

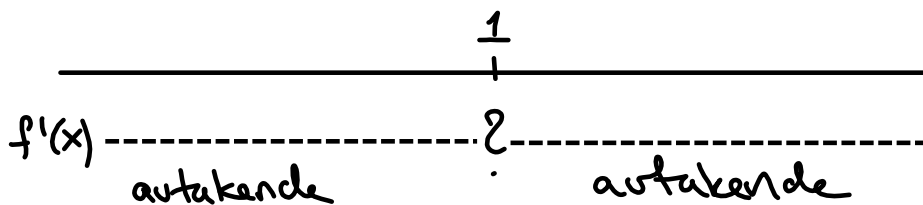
$1-3x^2 = 0$
 $1 = 3x^2$
 $\frac{1}{3} = x^2$
 $x = \pm \sqrt{\frac{1}{3}}$

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40) $f(x) = \begin{cases} -x+1 & \text{hvis } x \leq 1 \\ -x+2 & \text{hvis } x > 1 \end{cases}$ (ikke gjennomgått på kollektivum)



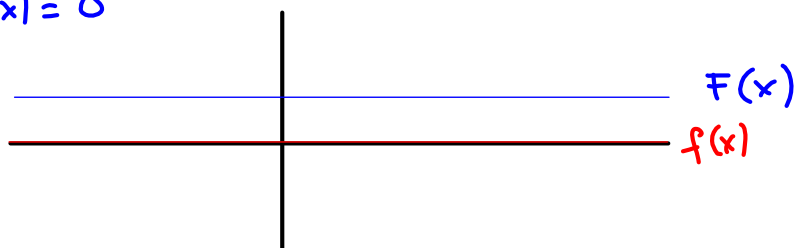
$$f'(x) = \begin{cases} -1 & x < 1 \\ -1 & x > 1 \end{cases}$$



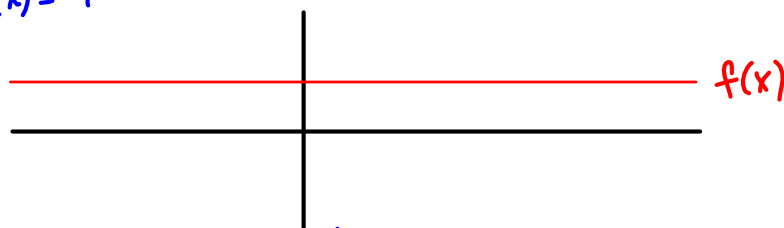
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41) skisser grafen til f
 skisser grafen til en funksjon $F(x)$ som har f som sin deriverte.

a) $f(x) = 0$



b) $f(x) = 1$



Tegn inn $F(x)$!

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