# Noen aktuelle formler

## Derivasjonsregler

**Spesielle:** $(x^n)' = nx^{n-1}$
$(a^x)' = a^x \ln a$ spesielt $(e^x)' = e^x$
$(\ln x)' = \frac{1}{x}$
$(\sin x)' = \cos x$ $(\cos x)' = -\sin x$
$(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$ $(\cot x)' = -\frac{1}{\sin^2 x}$

**Generelle:** $(f(x) + g(x))' = f'(x) + g'(x)$
$(f(x) \cdot g(x))' = f(x) \cdot g'(x) + f'(x) \cdot g(x)$
$(\frac{f(x)}{g(x)})' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$
$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

## Spesielle funksjoner

**Eksponensialfunksj.**: $a^x a^y = a^{x+y}$ $\frac{a^x}{a^y} = a^{x-y}$ $a^{-x} = \frac{1}{a^x}$ $(a^x)^y = a^{xy}$

**Logaritmer**: $\ln(xy) = \ln x + \ln y$ $\ln(\frac{x}{y}) = \ln x - \ln y$
$\ln \frac{1}{x} = -\ln x$ $\ln(x^a) = a \ln x$

**Trigonometriske**: $\sin^2 x + \cos^2 x = 1$
$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$\cos(x + y) = \cos x \cos y - \sin x \sin y$
$\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

**Eksakte verdier**:

| $v$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\sin v$ | 0 | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 |
| $\cos v$ | 1 | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | 0 |
| $\tan v$ | 0 | $\sqrt{3}/3$ | 1 | $\sqrt{3}$ | $-$ |

## Integrasjonsregler

**Spesielle:** $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$ $n \neq -1$ $\int \frac{1}{x} \, dx = \ln x + C,\ x > 0$
$\int a^x \, dx = \frac{1}{\ln a} a^x + C$ spesielt $\int e^x \, dx = e^x + C$
$\int \sin x \, dx = -\cos x + C$ $\int \cos x \, dx = \sin x + C$

**Generelle:** $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$

$\int f'(g(x))g'(x)\,dx = f(g(x)) + C$

**Bestemte integraler:** $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

## Differens- og differensialligninger

**Første ordens differensialligning**, $y' + f(x)y = g(x)$:

$$y(x) = e^{-\int f\,dx} \int e^{\int f\,dx} g(x)\,dx$$

**Andre ordens differensialligning**, $y'' + py' + qy = 0$:

$$y(x) = \begin{cases} Ce^{r_1 x} + De^{r_2 x} & \text{hvis to reelle røtter } r_1 \neq r_2 \\ Ce^{rx} + Dxe^{rx} & \text{hvis én reell rot } r \\ Ce^{ax}\cos(bx) + De^{ax}\sin(bx) & \text{hvis to komplekse røtter } r = a \pm ib \end{cases}$$

**Første ordens homogen differenslikning**;

$$x_n - kx_{n-1} = 0 \;\; : \quad x_n = Ck^n$$

**Andre ordens homogen differenslikning**;

$$x_{n+2} + bx_{n+1} + cx_n = 0, \qquad \text{karakteristisk polynom: } r^2 + br + c,$$

$$x_n = \begin{cases} Cr_1^n + Dr_2^n & \text{hvis to reelle røtter } r_1 \neq r_2 \\ Cr^n + Dnr^n & \text{hvis én reell rot } r \\ C\rho^n \cos(n\theta) + D\rho^n \sin(n\theta) & \text{hvis to komplekse røtter } r = \rho e^{\pm i\theta} \end{cases}$$

## Matriseregning og komplekse tall

**Komplekse tall**:

$$z = a + ib = r(\cos\theta + i\sin\theta) = re^{i\theta}, \quad r = \sqrt{a^2 + b^2}$$

$$\text{Realdel}: \; Re(z) = a, \quad \text{Imaginærdel}: \; Im(z) = b$$

$$\text{Kompleks konjugert : } \overline{z} = a - ib = re^{-i\theta}$$

$$\text{De Moivres formel : } \ (\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$$

**Matriseprodukt**:

$$(x_{ij}) \cdot (y_{ij}) = (z_{ij}), \quad , z_{ij} = \sum_{k=1}^{n} x_{ik} y_{kj}$$

**Determinanter**: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1$$

$$\det(AB) = \det(A)\det(B), \quad \det(A^{-1}) = \frac{1}{\det(A)}, \quad \det(A^T) = \det(A)$$

**Egenvektor/egenverdi**:

$$A\mathbf{x} = \lambda\mathbf{x}, \quad \text{karakteristisk polynom: } \det(\lambda I_n - A)$$

**Invers matrise**: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$