MAT 1001, fall 2016

Compulsory Exercise 2 (Oblig 2)

Deadline for hand-in: Thursday, November 10th, 2:30 PM

You are allowed to work together on the exercises, but everyone has to handle inn their own solution. The assignment must be handed in in the special box at the floor marked "7" in Niels Henrik Abels hus (math building) before the deadline. Remember to fill in and attach a front page - front pages are found nearby the box, or on the Internet. See *http://www.uio.no/studier/admin/obligatoriske-aktiviteter/mn-math-oblig.html* for further information.

Each exercise counts 0-5 points, all together a maximum of 40 points. Your Oblig 2 is approved if you score 20 points or more, and you have at least tried to solve all the exercises.

Exercise 1. ***(About hamonic functions, with an surprising answer (?))***

A harmonic function is given as a sum of three other harmonic functions

$$H(x) = \cos\left(\tfrac{2\pi}{3}x\right) + \cos\left(\tfrac{2\pi}{3}(x-1)\right) + \cos\left(\tfrac{2\pi}{3}(x-2)\right)$$

Write $H(x)$ as $A\cos(\omega(x - x_0))$ for some value of A, ω and x_0.

Exercise 2. ***(Repetition of 2nd order difference equations)***

a) A 2nd order difference equation is given by

$$x_{n+2} + x_{n+1} + x_n = 0$$

Find the general solution of this equation.

b) We consider the same difference equation as in exercise a), but in this exercise we shall vary the middle term in order to find other types of solutions.

$$x_{n+2} + b \cdot x_{n+1} + x_n = 0$$

Describe the solutions for different values of b. Two of the values of b will give only one root of the characteristic polynomial. Write up the general solution in these cases.

c) We go back to the equation of exercise a), but now we look at an inhomogenous equation

$$x_{n+2} + x_{n+1} + x_n = n^2 - n - 1$$

Find the solution of this equation which satisfies the initial condition $x_0 = 1$ and $x_1 = -1$.

Exercise 3. ***(Application of difference equations)***

We consider a simple model of a closed economy (without import and export) Gross domestic product (GDP) B_n in the year n is given by

$$B_n = I_n + C_n + G_n$$

where I_n denotes investments in physical capital, C_n is the personal consumption expenditures (PCE) and G_n is the public consumption, all measured at the year n. The model assumes that the public consumption is constant in time, i.e. $G_n = G$ for all n. We also assume that the PCE is proporsional to GDP for last year, i.e.

$$C_n = \alpha B_{n-1}$$

and that the investments are proportional to the increase in PCE from last year, i.e.

$$I_n = \beta(C_n - C_{n-1})$$

The constants α and β are positive real numbers.

a) Show that this model can be described by a difference equation

$$B_n - \alpha(\beta + 1)B_{n-1} + \alpha\beta B_{n-2} = G$$

b) If the expression under the root sign (the discriminant) in the solution of the characteristic equation is a negative number, then the solution of the difference equation will be given by a periodic function. Show that this is the case if

$$\alpha < \frac{4\beta}{(\beta + 1)^2}$$

Exercise 4. ***(Confusing integration constants!)***

The addition formula for sine of the double angle is given $\sin 2x = 2 \sin x \cos x$. Compute

$$\int \sin 2x \, dx \qquad \text{og} \qquad 2 \int \sin x \cos x \, dx$$

by using substitution and integration by parts respectively. The answers are apparently different. What is the secret?

Exercise 5. ***(Application of integration)***

A water tank is supplied with water with a net influx rate

$$v(t) = \frac{1}{\sqrt{2}} V_0 e^{-kt} \sqrt{1 + e^{-kt}}$$

The rate is a function of time t and V_0 and k are positive constants.

Give an interpretation of the constant V_0 and compute the indefinite integral

$$\int \frac{1}{\sqrt{2}} V_0 e^{-kt} \sqrt{1 + e^{-kt}} \, dt$$

THE END.