LØSNINGSFORSLAG OPPGAVER
6.+7. FEBRUAR

1

a)

$$\begin{bmatrix}1000 & 3000\end{bmatrix}\begin{bmatrix}0{,}2\\0{,}7\end{bmatrix} = 1000\cdot 0{,}2 + 3000\cdot 0{,}7$$
$$= \underline{\underline{2300}}$$

$$\begin{bmatrix}2000 & 1000 & 5000\end{bmatrix}\begin{bmatrix}0{,}3\\0{,}5\\0{,}65\end{bmatrix} = 2000\cdot 0{,}3 + 1000\cdot 0{,}5 + 5000\cdot 0{,}65$$
= Regn selv!

$$\begin{bmatrix}500 & 300 & 0 & 1000\end{bmatrix}\begin{bmatrix}0{,}1\\0{,}8\\0{,}4\\0{,}3\end{bmatrix} = 500\cdot 0{,}1 + 300\cdot 0{,}8 + 0\cdot 0{,}4 + 1000\cdot 0{,}3$$
= Regn selv!

b)

$$\begin{bmatrix}1000 & 3000\end{bmatrix}\begin{bmatrix}0{,}09 & -0{,}06\\-0{,}06 & 0{,}0625\end{bmatrix}\begin{bmatrix}1000\\3000\end{bmatrix}$$

$$= \begin{bmatrix}1000\cdot 0{,}09 + 3000\cdot(-0{,}06) & 1000\cdot(-0{,}06) + 3000\cdot 0{,}0625\end{bmatrix}\begin{bmatrix}1000\\3000\end{bmatrix}$$

$$= \begin{bmatrix}-90 & 127{,}5\end{bmatrix}\begin{bmatrix}1000\\3000\end{bmatrix} = (-90)\cdot 1000 + 127{,}5\cdot 3000$$
$$= \underline{\underline{292\,500}}$$

$$\begin{bmatrix} 2000 & 1500 & 3000 \end{bmatrix} \begin{bmatrix} 0{,}04 & -0{,}003 & 0{,}045 \\ -0{,}003 & 0{,}0225 & -0{,}01875 \\ 0{,}045 & -0{,}01875 & 0{,}0625 \end{bmatrix} \begin{bmatrix} 2000 \\ 1500 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 2000 \cdot 0{,}04 + 1500 \cdot (-0{,}003) + 3000 \cdot 0{,}045 & 2000\,(-0{,}003) + 1500 \cdot 0{,}0225 + 3000\,(-0{,}01875) & 2000\,(0{,}045) + 1500 \cdot (-0{,}01875) + 3000 \cdot 0{,}0625 \end{bmatrix} \begin{bmatrix} 2000 \\ 1500 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 210{,}5 & -28{,}5 & 249{,}375 \end{bmatrix} \begin{bmatrix} 2000 \\ 1500 \\ 3000 \end{bmatrix}$$

$$= 210{,}5 \cdot 2000 + (-28{,}5) \cdot 1500 + 249{,}375 \cdot 3000$$

$$= \underline{\underline{1\,126\,375}}$$

c) 2x2-matrize

$$\begin{bmatrix} 0{,}09 & -0{,}06 \\ -0{,}06 & 0{,}0625 \end{bmatrix}$$

$$\hat{\sigma}_1^2 = 0{,}09 \Rightarrow \underline{\underline{\hat{\sigma}_1 = 0{,}3}}$$

$$\hat{\sigma}_2^2 = 0{,}0625 \Rightarrow \underline{\underline{\hat{\sigma}_2 = 0{,}25}}$$

$$\hat{c}_{1,2} = -0{,}06 = \hat{\rho}_{1,2} \cdot \hat{\sigma}_1 \, \hat{\sigma}_2$$

$$\Rightarrow \underline{\underline{\hat{\rho}_{1,2}}} = \frac{-0{,}06}{0{,}3 \cdot 0{,}25} = \underline{\underline{-0{,}8}}$$

3x3 matrix

$$\begin{bmatrix} 0{,}04 & -0{,}003 & 0{,}045 \\ -0{,}003 & 0{,}0225 & -0{,}01875 \\ 0{,}045 & -0{,}01875 & 0{,}0625 \end{bmatrix} \Rightarrow \begin{array}{l} \underline{\underline{\hat{\sigma}_1}} = \sqrt{0{,}04} = \underline{\underline{0{,}2}} \\ \underline{\underline{\hat{\sigma}_2}} = \sqrt{0{,}0225} = \underline{\underline{0{,}15}} \\ \underline{\underline{\hat{\sigma}_3}} = \sqrt{0{,}0625} = \underline{\underline{0{,}25}} \end{array}$$

$$\hat{C}_{1,2} = -0{,}003 = \hat{\rho}_{1,2} \cdot \hat{\sigma}_1 \hat{\sigma}_2 \Rightarrow \underline{\underline{\hat{\rho}_{1,2}}} = \frac{-0{,}003}{0{,}2 \cdot 0{,}15} = \underline{\underline{-0{,}1}}$$

$$\hat{C}_{1,3} = 0{,}045 = \hat{\rho}_{1,3} \hat{\sigma}_1 \hat{\sigma}_3 \Rightarrow \underline{\underline{\hat{\rho}_{1,3}}} = \frac{0{,}045}{0{,}2 \cdot 0{,}25} = \underline{\underline{0{,}9}}$$

$$\hat{C}_{2,3} = -0{,}01875 = \hat{\rho}_{2,3} \cdot \hat{\sigma}_2 \cdot \hat{\sigma}_3 \Rightarrow \underline{\underline{\hat{\rho}_{2,3}}} = \frac{-0{,}01875}{0{,}15 \cdot 0{,}25} = \underline{\underline{-0{,}5}}$$

$\hat{\sigma}_R = \sqrt{292500} \simeq 541$ MWh i 2x2-tilfellet.

$\hat{\sigma}_R = \sqrt{1126375} \simeq 1061$ MWh i 3x3-tilfellet

$$\begin{bmatrix} 3000 & 1000 \end{bmatrix} \begin{bmatrix} 0{,}09 & -0{,}06 \\ -0{,}06 & 0{,}0625 \end{bmatrix} \begin{bmatrix} 3000 \\ 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 210 & -117{,}5 \end{bmatrix} \begin{bmatrix} 3000 \\ 1000 \end{bmatrix} = 512\,500$$

Produksjonens standardavvik blir

$$\underline{\hat{\sigma}_R} = \sqrt{512\,500} \approx \underline{\underline{716 \text{ MWh}}}.$$

Observer at hvis du bygger 1000 MW i lokasjon 1 og 7000 MW i lokasjon 2, får du et lavere standardavvik på produksjonen (541 MWh) enn hvis du bygger 3000 MW i lokasjon 1 og 1000 MW i lokasjon 2.

Ikke overraskende vil variasjonen i produksjonen avhenge av hvor mye du installerer i hver lokasjon, og noen valg gir mindre variasjon enn andre.

2

a)

$$\begin{bmatrix} \hat{\sigma}_1^2 & \hat{c}_{1,2} & \hat{c}_{1,3} \\ \hat{c}_{1,2} & \hat{\sigma}_2^2 & \hat{c}_{2,3} \\ \hat{c}_{1,3} & \hat{c}_{2,3} & \hat{\sigma}_3^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} w_1 \hat{\sigma}_1^2 + w_2 \hat{c}_{1,2} + w_3 \hat{c}_{1,3} \\ w_1 \hat{c}_{1,2} + w_2 \hat{\sigma}_2^2 + w_3 \hat{c}_{2,3} \\ w_1 \hat{c}_{1,3} + w_2 \hat{c}_{2,3} + w_3 \hat{\sigma}_3^2 \end{bmatrix}$$

b)

$$\underline{C_w} = \begin{bmatrix} 2000 \cdot 0{,}04 + 1500 \cdot (-0{,}003) + 3000 \cdot 0{,}045 \\ 2000 \cdot (-0{,}003) + 1500 \cdot 0{,}0225 + 3000 \cdot (-0{,}01875) \\ 2000 \cdot 0{,}045 + 1500 \cdot (-0{,}01875) + 3000 \cdot 0{,}0625 \end{bmatrix}$$

$$= \begin{bmatrix} 210{,}5 \\ -28{,}5 \\ 249{,}375 \end{bmatrix}$$

$$\underline{w}^T C \underline{w} = \begin{bmatrix} 2000 & 1500 & 3000 \end{bmatrix} \begin{bmatrix} 210{,}5 \\ -28{,}5 \\ 249{,}375 \end{bmatrix}$$

$$= 2000 \cdot 210{,}5 + 1500 \cdot (-28{,}5) + 3000 \cdot 249{,}375$$

$$= \underline{\underline{1\,126\,375}}$$

Sammenlign med hva vi fant i oppg 1b, 3x3-tilfellet. Det er det samme!

c) Fra a) finner vi.

$$\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} w_1 \hat{\sigma}_1^2 + w_2 \hat{c}_{1,2} + w_3 \hat{c}_{1,3} \\ w_1 \hat{c}_{1,2} + w_2 \hat{\sigma}_2^2 + w_3 \hat{c}_{2,3} \\ w_1 \hat{c}_{1,3} + w_2 \hat{c}_{2,3} + w_3 \hat{\sigma}_3^2 \end{bmatrix}$$

$$= w_1 \left(w_1 \hat{\sigma}_1^2 + w_2 \hat{c}_{1,2} + w_3 \hat{c}_{1,3} \right) + w_2 \left(w_1 \hat{c}_{1,2} + w_2 \hat{\sigma}_2^2 + w_3 \hat{c}_{2,3} \right)$$

$$+ w_3 \left(w_1 \hat{c}_{1,3} + w_2 \hat{c}_{2,3} + w_3 \hat{\sigma}_3^2 \right)$$

$$= w_1^2\hat{\sigma}_1^2 + 2w_1w_2\hat{c}_{1,2} + 2w_1w_3\hat{c}_{1,3} + w_2^2\hat{\sigma}_2^2$$

$$+ 2w_2w_3\hat{c}_{2,3} + w_3^2\hat{\sigma}_3^2$$

$$\underline{w}^T C = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} \hat{\sigma}_1^2 & \hat{c}_{1,2} & \hat{c}_{1,3} \\ \hat{c}_{1,2} & \hat{\sigma}_2^2 & \hat{c}_{2,3} \\ \hat{c}_{1,3} & \hat{c}_{2,3} & \hat{\sigma}_3^2 \end{bmatrix}$$

$$= \Big[\underbrace{w_1\hat{\sigma}_1^2 + w_2\hat{c}_{1,2} + w_3\hat{c}_{1,3} \quad w_1\hat{c}_{1,2} + w_2\hat{\sigma}_2^2 + w_3\hat{c}_{2,3} \quad w_1\hat{c}_{1,3} + w_2\hat{c}_{2,3} + w_3\hat{\sigma}_3^2}\Big]$$

$$(\underline{w}^T C)\underline{w} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$= w_1\left(w_1\hat{\sigma}_1^2 + w_2\hat{c}_{1,2} + w_3\hat{c}_{1,3}\right) + w_2\left(w_1\hat{c}_{1,2} + w_2\hat{\sigma}_2^2 + w_3\hat{c}_{2,3}\right)$$

$$+ w_3\left(w_1\hat{c}_{1,3} + w_2\hat{c}_{2,3} + w_3\hat{\sigma}_3^2\right)$$

Ganger vi ut, får vi det samme! Generelt kalkulerer vi med at

$$(\underline{w}^T C)w = \underline{w}^T(C\underline{w})$$

3

a)

$$\begin{aligned} w_1 + 2w_2 &= 3 \quad \Big| \cdot (-2) \\ 2w_1 - 3w_2 &= 2 \end{aligned} \Rightarrow \begin{aligned} -2w_1 - 4w_2 &= -6 \\ 2w_1 - 3w_2 &= 2 \\ \hline -7w_2 &= -4 \end{aligned}$$

$$\Rightarrow \underline{\underline{w_2 = \frac{4}{7}}} \qquad w_1 + 2w_2 = 3 \Rightarrow \underline{\underline{w_1}} = 3 - 2w_2 = 3 - 2 \cdot \frac{4}{7} = \underline{\underline{\frac{13}{7}}}$$

b)

$$\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1 + 2w_2 \\ 2w_1 - 3w_2 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}}}$$

c)

$$\begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{7} \cdot 3 + \frac{2}{7} \cdot 2 \\ \frac{2}{7} \cdot 3 + \left(-\frac{1}{7}\right) \cdot 2 \end{bmatrix} = \begin{bmatrix} \frac{13}{7} \\ \frac{4}{7} \end{bmatrix} \begin{matrix} = w_1 \\ = w_2 \end{matrix}$$

Dette er ikke en del av oppgaven, men merk at

$$\begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} \frac{3}{7} \cdot 1 + \frac{2}{7} \cdot 2 & \frac{3}{7} \cdot 2 + \frac{2}{7} \cdot (-3) \\ \frac{2}{7} \cdot 1 + \left(-\frac{1}{7}\right) \cdot 2 & \frac{2}{7} \cdot 2 + \left(-\frac{1}{7}\right)(-3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Dermed har vi:

$$\underbrace{\begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}} = \begin{bmatrix} \frac{13}{7} \\ \frac{4}{7} \end{bmatrix}$$

$$\underline{\underline{\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{13}{7} \\ \frac{4}{7} \end{bmatrix}}}$$

oppgave 4 er en del av den obligatoriske oppgaven.