FASIT

1) $f = x^2y - x^2 - y^4 + 8y^2$

$\frac{\partial f}{\partial x} = 2xy - 2x = 2x(y-1) = 0 \iff x = 0$ eller $y = 1$

$\frac{\partial f}{\partial y} = x^2 - 4y^3 + 16y = 0$

Hvis $x = 0$ får vi $0 = -4y^3 + 16y = -4y(y^2-4) \iff y = 0, \pm 2$

Hvis $y = 1$ får vi $x^2 + 12 = 0$, ingen løsning.

Kritiske punkter $(0,0), (0,2), (0,-2)$

$\frac{\partial^2 f}{\partial x^2} = 2y - 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 2x \qquad \frac{\partial^2 f}{\partial y^2} = -12y^2 + 16$

Punkt	$A = \frac{\partial^2 f}{\partial x^2}$	$B = \frac{\partial^2 f}{\partial x \partial y}$	$C = \frac{\partial^2 f}{\partial y^2}$	$H = AC - B^2$	Type
$(0,0)$	-2	0	16	-32	Sadelpunkt
$(0,2)$	2	0	-32	-64	Sadelpunkt
$(0,-2)$	-6	0	-32	192	Lokalt maks.

2) a) $\vec{F} = (2x + y^2, 2xy)$

curl $\vec{F} = \frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial y}(2x + y^2) = 2y - 2y = 0$, så $\vec{F}$ er konservativt

siden $\vec{F}$ også er definert for alle (x,y).

$\frac{\partial f}{\partial x} = 2x + y^2 \Rightarrow f = x^2 + xy^2 + g(y)$

$\frac{\partial f}{\partial y} = 2xy + g'(y) = 2xy.$ Kan velge $g = 0$.

$\underline{f = x^2 + xy^2}$ er en potensialfunksjon.

b) $\int_C \vec{F} \cdot d\vec{r} = f(\vec{r}(1)) - f(\vec{r}(0)) = f(1, \sqrt{5}) - f(0,0)$

$= 1 + 1 \cdot 5 = \underline{\underline{6}}$

c) $\vec{r}\,'(t) = (2t, 3\sqrt{5}\, t^2)$

$ds = \|\vec{r}\,'(t)\| dt = \sqrt{4t^2 + 45t^4}\, dt = t\sqrt{4 + 45t^2}\, dt$

$L = \int_C ds = \int_0^1 t\sqrt{4 + 45t^2}\, dt = \frac{1}{90}\int_4^{49} u^{1/2}\, du = \frac{1}{90} \cdot \frac{2}{3}\left[u^{3/2}\right]_4^{49}$

$$\boxed{\begin{array}{l} u = 4 + 45t^2 \\ du = 90t\,dt \\ u(0) = 4,\ u(1) = 49 \end{array}}$$

$= \frac{1}{135} \cdot \left(49^{3/2} - 4^{3/2}\right) = \frac{1}{135}(7^3 - 2^3)$

$= \frac{1}{135}(343 - 8) = \frac{335}{135} = \underline{\underline{\frac{67}{27}}} \quad (\cong 2.481)$

d) $\Delta t = \frac{1}{4} = 0.25$

t	0	0.25	0.5	0.75	1
$f(t) = t\sqrt{4+45t^2}$	0	0.653	1.953	4.061	7

$L \cong \frac{0.25}{3}\left(0 + 4 \cdot 0.653 + 2 \cdot 1.953 + 4 \cdot 4.061 + 7\right) = \underline{2.480}$

3) $\frac{dy}{dt} = 2e^{-y}t$

Separeren variable

$e^{y} dy = 2t\, dt$

Integrerer hver side

$$e^y = t^2 + C$$

$y(0)=0$ gir $C = e^{y(0)} = e^0 = 1$, dvs $e^y = t^2+1$ og dermed

$$\underline{\underline{y = \ln(t^2+1)}}$$

4) Lagranges ligningen er $\nabla f(x,y) = \lambda \nabla g(x,y)$ med $g(x,y) = x^4+y^4$. Vi får

$$\begin{aligned} 8 &= 4\lambda x^3 \\ 1 &= 4\lambda y^3 \end{aligned} \Rightarrow x^3 = 8y^3, \quad x = 2y.$$

Vi setter inn:

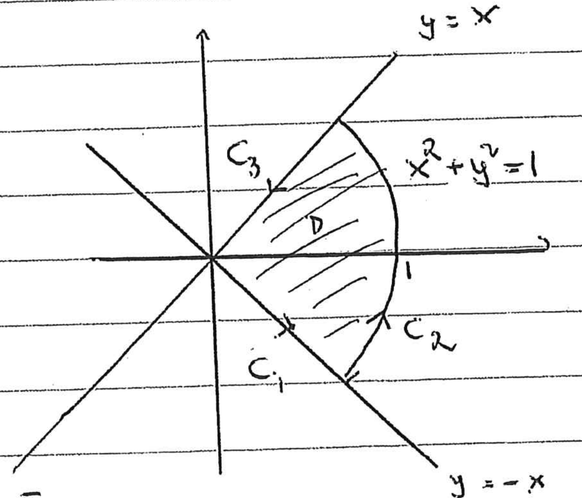
$$x^4+y^4 = (2y)^4 + y^4 = 17y^4 = 1, \text{ dvs } y = \pm \frac{1}{\sqrt[4]{17}}$$

og $x = 2y$

$$f\left(\frac{2}{\sqrt[4]{17}}, \frac{1}{\sqrt[4]{17}}\right) = \frac{16}{\sqrt[4]{17}} + \frac{1}{\sqrt[4]{17}} = \frac{17}{17^{1/4}} = \underline{17^{3/4}} \quad \text{Maks.}$$

$$f\left(-\frac{2}{\sqrt[4]{17}}, -\frac{1}{\sqrt[4]{17}}\right) = \underline{-17^{3/4}} \quad \text{Min.}$$

5)



a) I polarkoordinater er D gitt ved $0 \le r \le 1$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$.

$$A(D) = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$

$$\bar{x}\cdot A(D) = \iint_D x \, dA = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\int_0^1 r\cos\theta \, r \, dr \right) d\theta =$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{3}\cos\theta \, d\theta = \frac{1}{3}\left[\sin\theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{3}\left(\frac{1}{2}\sqrt{2} - \left(-\frac{1}{2}\sqrt{2}\right)\right) = \frac{\sqrt{2}}{3} \Rightarrow \bar{x} = \frac{4}{\pi}\cdot\frac{\sqrt{2}}{3} = \underline{\underline{\frac{4\sqrt{2}}{3\pi}}}$$

Av symmetrigrunner er $\bar{y} = 0$.

Tyngdepunkt : $\underline{\underline{\left(\frac{4\sqrt{2}}{3\pi}, 0\right)}}$

b) Vi har $\partial D = C_1 \cup C_2 \cup C_3$. $\vec{F}(x,y) = (-y, x)$ står vinkelrett på C_1 og C_3. Derfor er

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_3} \vec{F} \cdot d\vec{r} = 0.$$

C_2 er parameterisert $\vec{r}(t) = (\cos t, \sin t)$, $-\frac{\pi}{4} \leq t \leq \frac{\pi}{4}$

$$\int_{C} \vec{F} \cdot d\vec{r} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (-\sin t, \cos t) \cdot (-\sin t, \cos t)\, dt$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin^2 t + \cos^2 t)\, dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, dt = \underline{\underline{\frac{\pi}{2}}}$$

Dette gir $\oint_{\partial D} \vec{F} \cdot d\vec{r} = \frac{\pi}{2}$.

Green gir

$$\oint_{\partial D} \vec{F} \cdot d\vec{r} = \iint_D \operatorname{curl} \vec{F}\, dA = \iint_D (1-(-1))\, dA = \iint_D 2\, dA = 2A(D) = \underline{\underline{\frac{\pi}{2}}}$$

6 a) Karakteristiske ligning

$$\lambda^2 - (a+d)\lambda + (ad - bc) = \lambda^2 - (2+4)\lambda + (2 \cdot 4 - 3 \cdot 1)$$

$$= \lambda^2 - 6\lambda + 5 = 0$$

$$\lambda = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2} = \begin{cases} 5 \\ 1 \end{cases}$$

Generell løsning

$$x = Ce^{5t} + De^{t}$$

$$y = \frac{\lambda_1 - a}{b} Ce^{\lambda_1 t} + \frac{\lambda_2 - a}{b} De^{\lambda_2 t} = \frac{5-2}{1} Ce^{5t} + \frac{1-2}{1} De^{t}$$

$$= 3Ce^{5t} - De^{t}$$

$$\left.\begin{array}{l} x(0) = C + D = 3 \\ y(0) = 3C - D = 5 \end{array}\right\} \Rightarrow C = 2,\ D = 1$$

Løsning $\quad x = 2e^{5t} + e^{t}$

$\qquad\qquad \underline{\underline{y = 6e^{5t} - e^{t}}}$

Siden $\lambda_1 > \lambda_2 > 0$, har vi <u>frastøtende likevektspunkt.</u>

b) <u>Likevektspunkt</u>

$$\begin{array}{l} -3x - 2y + 7 = 0 \\ x - y + 1 = 0 \end{array} \Leftrightarrow \begin{array}{ll} 3x + 2y = 7 & (I) \\ x - y = -1 & (II) \end{array}$$

$(I) + 2(II)$: $5x = 5$, $x = 1$, $y = x + 1 = 2$

Likevektspunkt $\underline{\underline{(1, 2)}}$

Karakteristisk ligning:

$$\lambda^2 - (-3-1)\lambda + ((-3)(-1) - 1\cdot(-2)) = \lambda^2 + 4\lambda + 5 = 0$$

$$\lambda = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i.$$

Siden $\alpha = -2 < 0$ har vi <u>tiltrekkende likevektspunkt.</u>

(Spiral)