



2)

a)  $M = \begin{pmatrix} 0.3 & 0.4 & 0.8 \\ 0.4 & 0.5 & 0 \\ 0.3 & 0.1 & 0.2 \end{pmatrix}$

b)  $\vec{x}_0 = \begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix}_{3 \times 3 \rightarrow 3 \times 1}$

$$\vec{x}_1 = M \cdot \vec{x}_0$$

$$= \begin{pmatrix} 0.3 & 0.4 & 0.8 \\ 0.4 & 0.5 & 0 \\ 0.3 & 0.1 & 0.2 \end{pmatrix} \begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3 \cdot 100 + 0.4 \cdot 100 + 0.8 \cdot 100 \\ \vdots \\ 150 \\ 90 \\ 60 \end{pmatrix}$$

c)  $M \cdot M$

$$\begin{pmatrix} 0.3 & 0.4 & 0.8 \\ 0.4 & 0.5 & 0 \\ 0.3 & 0.1 & 0.2 \end{pmatrix} \cdot \begin{pmatrix} 0.3 & 0.4 & 0.8 \\ 0.4 & 0.5 & 0 \\ 0.3 & 0.1 & 0.2 \end{pmatrix}$$

$$= 0.3 \cdot 0.3 + 0.4 \cdot 0.4 + 0.8 \cdot 0.3$$

$$= 0.49$$

$$0.3 \cdot 0.4 + 0.4 \cdot 0.5 + 0.8 \cdot 0$$

$$= 0.4$$

$$\vdots$$

$$M^2 = \begin{pmatrix} 0.49 & 0.4 & 0.4 \\ 0.32 & 0.41 & 0.32 \\ 0.19 & 0.19 & 0.28 \end{pmatrix}$$

$$\vec{x}_2 = M \vec{x}_1 = M \cdot (M \vec{x}_0) = M^2 \vec{x}_0$$

$$\vec{x}_2 = \begin{pmatrix} 0.49 & 0.4 & 0.4 \\ 0.32 & 0.41 & 0.32 \\ 0.19 & 0.19 & 0.28 \end{pmatrix} \begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} 0.49 \cdot 100 + 0.4 \cdot 100 + 0.4 \cdot 100 \\ \vdots \\ 129 \\ 105 \\ 66 \end{pmatrix}$$

d)  $\vec{x} = M \cdot \vec{x}_1; \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$x_1 + x_2 + x_3 = 91$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.3 & 0.4 & 0.8 \\ 0.4 & 0.5 & 0 \\ 0.3 & 0.1 & 0.2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

I:  $x_1 = 0.3x_1 + 0.4x_2 + 0.8x_3$   
II:  $x_2 = 0.4x_1 + 0.5x_2$   
III:  $x_3 = 0.3x_1 + 0.1x_2 + 0.2x_3$

Oppr om II

$$x_2 = 0.4x_1 + 0.5x_2$$

$$\frac{0.5x_2}{0.5} = \frac{0.4x_1}{0.5} + x_2 (x_2 - 0.5x_2 = 0.5x_1)$$

III:  $x_2 = 0.8x_1$   
Sett inn i: I & III

I:  $x_1 = 0.3x_1 + 0.4(0.8x_1) + 0.8x_3$   
III:  $x_3 = 0.3x_1 + 0.1(0.8x_1) + 0.2x_3$

I:  $-x_1 + 0.3x_1 + 0.32x_1 + 0.8x_3 = 0$   
III:  $0.3x_1 + 0.08x_1 + 0.2x_3 - x_3 = 0$

I:  $-0.38x_1 + 0.8x_3 = 0$   
III:  $0.38x_1 - 0.8x_3 = 0$   
I = -III

I:  $0.8x_3 = 0.38x_1$   
 $\frac{x_3}{0.8} = \frac{0.38x_1}{0.8}$   
 $x_3 = 0.475x_1$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0.8x_1 \\ 0.475x_1 \end{pmatrix}$$

NB:  $x_1 + x_2 + x_3 = 91$  (Betingelse)  
 $x_1 + 0.8x_1 + 0.475x_1 = 91$   
 $\frac{2.275x_1}{2.275} = \frac{91}{2.275}$   
 $\vec{x} = \begin{pmatrix} 40 \\ 32 \\ 19 \end{pmatrix} \quad x_1 = 40$