Mock Exam MAT 1100, Fall 03

This Mock Exam has the same format as the real exam and contains problems of the same kind and difficulty. The first part of the exam consists of 10 multiple choice questions each counting 3 points. There is only one correct alternative for each question. If you answer a question incorrectly or not at all, you get zero points. Hence there is no "penalty" for guessing. The second part of the exam consists of more traditional problems. In this part each of the 7 questions is worth 10 points. The total score is thus 100 points. In the second part of the exam you must explain how you have reached your answers. Unsubstantiated answers will receive 0 points even when they are correct!

CALCULATORS ARE THE ONLY ALLOWED TOOLS ON THE EXAM. THE SHEET WITH FORMULAS WILL BE AVAILABLE AS PART OF THE PROBLEM SET.

PART 1: MULTIPLE CHOICE

1. The integral $\int \frac{\cos x}{1+\sin^2 x}\, dx$ equals:

☐ $\ln(1+\sin^2 x) + C$

☐ $\cot(1+\sin^2 x) + C$

☐ $\arctan(\sin x) + C$

☐ $\arccos(\sin x) + C$

☐ $-\frac{1}{1+\sin x} + C$

2. If $a > 0$ is a constant, then $\int_0^2 x^{a-1}\, e^{x^a}\, dx$ equals:

☐ $\frac{1}{a}\left(e^{2^a} - 1\right)$

☐ $2^a e^{2^a}$

☐ $\frac{1}{a}\left(e^{2a} - 1\right)$

☐ the integral diverges

☐ $\frac{e^{2^a}}{a}$

3. If we want to decompose $\frac{x^2+4x+5}{(x+1)(x^2+2x+5)^2}$ into partial fractions, we should look for an expression of the form:

☐ $\frac{Ax+B}{x+1} + \frac{Bx+C}{x^2+2x+5}$

☐ $\frac{A}{x+1} + \frac{Bx+C}{x^2+2x+5}$

☐ $\frac{A}{x+1} + \frac{Bx+C}{(x^2+2x+5)^2}$

☐ $\frac{A}{x+1} + \frac{B}{x^2+2x+5} + \frac{C}{(x^2+2x+5)^2}$

☐ $\frac{A}{x+1} + \frac{Bx+C}{x^2+2x+5} + \frac{Dx+E}{(x^2+2x+5)^2}$

4. When we substitute $u = \sqrt{x}+1$ in the integral $\int_1^9 \arctan(\sqrt{x}+1)\,dx$, we get:

☐ $\int_1^9 2(u-1)\arctan u\,du$
☐ $\int_2^4 \arctan u\,du$
☐ $\int_2^4 2(u-1)\arctan u\,du$
☐ $\int_1^9 \arctan u\,du$
☐ $\int_2^4 \frac{1}{2\sqrt{u}}\arctan u\,du$

5. The improper integral $\int_e^\infty \frac{1}{x\ln x}\,dx$:

☐ diverges
☐ converges and equals $\frac{\pi}{2}$
☐ converges and equals 2
☐ converges and equals $\sqrt{5}$
☐ converges and equals 4

6. The derivative of the function $F(x) = \int_0^{\sqrt{x}} e^{t^2}\,dt$, $x > 0$, equals:

☐ does not exixt since it is impossible to compute the integral
☐ e^x
☐ e^{x^2}
☐ does not exist since the integral diverges
☐ $\frac{e^x}{2\sqrt{x}}$

7. The gradient of $f(x,y) = x^2e^{-xy}$ is:

☐ $(2xe^{-xy} - x^2ye^{-xy}, 2xe^{-xy} - x^3e^{-xy})$
☐ $(-2xye^{-xy}, -x^3e^{-xy})$
☐ $(2xe^{-xy}, x^2e^{-xy})$
☐ $(2xe^{-xy} - x^2ye^{-xy}, -x^3e^{-xy})$
☐ $(2xe^{-xy} - x^2ye^{-xy}, x^2e^{-xy})$

8. If $f(x,y) = 2xy+y^2$, $\mathbf{a} = (1,2)$ and $\mathbf{r} = (3,-1)$ the directional derivative $f'(\mathbf{a},\mathbf{r})$ equals:

☐ $\frac{1}{2}$
☐ -4
☐ 3
☐ 6
☐ $-\frac{17}{4}$

9. When we are standing at the point (1,-3), the function $f(x,y) = 3x^2y+xy$ is increasing most rapidly in the direction of:

☐ $(1,2)$
☐ $(-18,4)$
☐ $(3,-4)$
☐ $(-21,4)$
☐ $(-7,1)$

10. The limit $\lim_{(x,y)\to(0,0)} \frac{x^2+3xy}{\sqrt{x^2+y^2}}$ equals:

☐ 0
☐ 2
☐ ∞
☐ does not exist as we get different answers when we approach zero from different directions
☐ $-\frac{1}{2}$

PART 2

Remember that in this part you have to substantiate your answers

Problem I. Find the square roots of the complex number $z = -2 + 2i\sqrt{3}$.

Problem II. Solve the integral $\int x \ln(x+1)\, dx$.

Problem III. The function f is given by $f(x, y) = x^3 + 5x^2 + 3y^2 - 6xy$.
a) Find the stationary points of f.
b) Decide whether the stationary points are local minima, local maxima or saddle points.

Problem IV.
a) Let a be a number between 0 and 5. The area bounded by the x-axis, the y-axis, the graph of the function $f(x) = \sqrt{25 - x^2}$ and the line $x = a$ is rotated around the x-axis. Find the volume of the solid of revolution expressed in terms of a.
b) A spherical tank of radius 5 meters is being emptied of water. When the water level in the tank is 2 meters, the tank is emptied at a speed of 0.5 cubic meters per minute. How fast is the water level decreasing at this moment?

Problem V. A function f of one variable is called a *Lipschitz function* on the interval I if there is a number K such that $|f(x) - f(y)| \leq K|x - y|$ for all $x, y \in I$. Show first that if f is a Lipschitz function on the interval I, then f is continuous on I. Then prove the following statement:

"If the derivative g' is continuous on a closed, bounded interval I, then g is a Lipschitz function on I."

Good luck!