Mandatory assignment: MAT1100, Fall 2005.
Deadline: September 16, 2005.

Show all your work. Exercise 4 is optional. Good luck!

Exercise 1. Complex numbers:
a) Find the polar form of $\sqrt{3} + i$.
b) Express

$$\frac{(\sqrt{3}+i)^{15}}{(1-i)^{29}}$$

in $a + ib$ form for real numbers $a, b \in \mathbb{R}$.
c) Find all complex solutions to $z^5 = 1$.
d) Let z_1 and $z_2 \neq 0$ be complex numbers. Prove that $z_1\overline{z_2}$ is real and positive if and only if $z_1 = rz_2$ for some positive real number $r \in \mathbb{R}$.

Exercise 2. Sequences, continuous functions:
a) Let $n \geq 1$ and

$$a_n = \frac{en+1}{n}.$$

Prove that the sequence $\{a_n\}$ converges to e.
b) Use an $\epsilon - \delta$-argument to prove

$$f(x) = x^2$$

is a continuous function.
c) Define f by

$$f(x) = \begin{cases} x \sin \frac{1}{x} - \cos \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$$

Is f continuous at 0 ?
d) Let $a > 0$ be a constant. Evaluate

$$\lim_{x \to \infty} \frac{a^x}{1 + a^x}.$$

Exercise 3. Differentiable functions:
a) Find the derivative of

$$f(x) = \ln(\frac{x^2}{1+x^2}).$$

b) Find $f'(x)$ when

$$e^{f(x)} = 1 + x^2.$$

Exercise 4. Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a function such that for every $y \in \mathbb{R}$, the equation $f(x) = y$ has exactly two distinct solutions. Prove that f cannot be continuous.