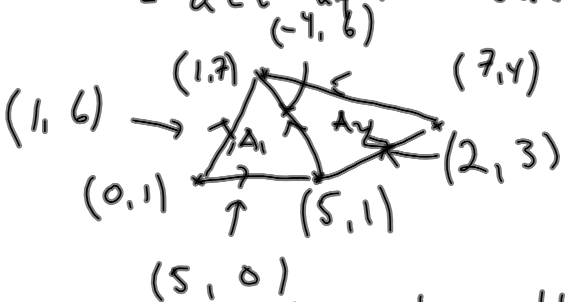
1.8   1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 17, 20

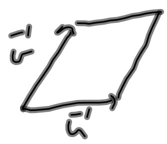
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} =$$

$$= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= aei - afh - bdi + bfg + cdh - ceg$$

3)





$A = |\vec{u} \times \vec{v}|$

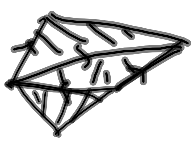
$\vec{u} = (a, b, 0)$
$\vec{v} = (c, d, 0)$

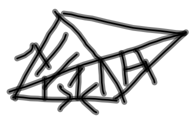
$\Rightarrow A = \left| \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right|$

$= |ad - bc|$

$$A = A_1 + A_2 = \frac{1}{2}\left|\begin{vmatrix} 1 & 6 \\ 5 & 0 \end{vmatrix}\right| + \frac{1}{2}\left|\begin{vmatrix} 2 & 3 \\ -4 & 6 \end{vmatrix}\right|$$

$$= 15 + 12 = \underline{27}$$





6 $\quad \det(\vec{a}, \vec{b}) = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \qquad \vec{a} = (a_1, a_2)$

$\qquad \vec{b} = (b_1, b_2)$

$= 0$ hvis og bare hvis

$\vec{a} = \vec{0}$ eller $\vec{b} = \vec{0}$ eller

$\vec{a}$ og $\vec{b}$ er parallelle.

Vil vise $\Longrightarrow$

$\det(\vec{a}, \vec{b}) = a_1 b_2 - a_2 b_1 = 0 \Rightarrow a_1 b_2 = a_2 b_1$

Anta at $a_2 b_2 \neq 0$ da er $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

så $\underline{\vec{a} \text{ og } \vec{b} \text{ er parallelle.}}$

hvis $a_2 b_2 = 0$ så er $a_2 = 0$ eller $b_2 = 0$

$a_2 = 0 \quad b_2 \neq 0 \Rightarrow a_1 b_2 = a_2 b_1 = 0$

så $a_1 = 0$ og dermed $\underline{\vec{a} = 0}$

$a_2 = 0 \quad b_2 = 0 \Rightarrow \vec{a} = (a_1, 0) \quad \vec{b} = (b_1, 0)$

så $\underline{\vec{a} \text{ og } \vec{b} \text{ er parallelle.}}$

$a_2 \neq 0 \quad b_2 = 0 \Rightarrow b_1 = 0 \Rightarrow \underline{\vec{b} = 0}$

Dermed er retningen $\Longrightarrow$ vist

$\Longleftarrow \quad \vec{a}, \vec{b}$ parallelle $\Rightarrow \vec{a} = \lambda \vec{b}$

$\Rightarrow (a_1, a_2) = (\lambda b_1, \lambda b_2) \Rightarrow \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0$

$\vec{a} = 0 \Rightarrow \begin{vmatrix} 0 & 0 \\ b_1 & b_2 \end{vmatrix} = 0$

$\vec{b} = 0 \Rightarrow \begin{vmatrix} a_1 & a_2 \\ 0, & 0 \end{vmatrix} = 0$

$\Box$

9. $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

$$\underbrace{\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}}_{= A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$A^{-1} \cdot A \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$ax = b$$
$$x = \frac{b}{a}$$
$$\underbrace{a^{-1}a}_{=1} x = a^{-1} b$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{a_1 b_2 - a_2 b_1}\begin{pmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

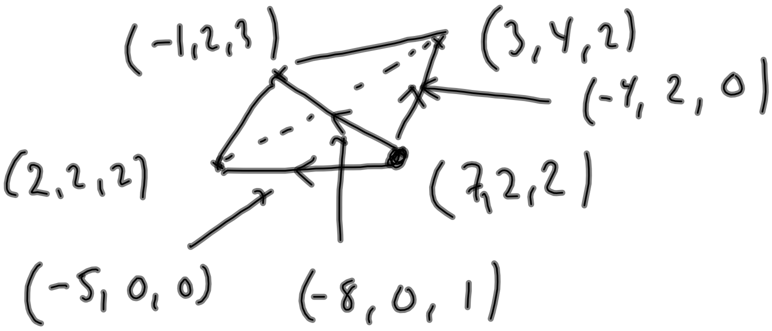
$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{a_1 b_2 - a_2 b_1}\begin{pmatrix} b_2 c_1 - b_1 c_2 \\ -a_2 c_1 + a_1 c_2 \end{pmatrix}$$

$$= \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}\begin{pmatrix} \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \\ \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \end{pmatrix}$$

b) $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0 \Rightarrow$ ingen eller uendelig mange løsninger.

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

12



$$V = \left| (\vec{v} \times \vec{w}) \cdot \vec{u} \right| = \left| \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \right|$$

$\vec{u} = (a_1, a_2, a_3)$

$\vec{v} = (b_1, b_2, b_3)$

$\vec{w} = (c_1, c_2, c_3)$

$$V = \frac{1}{3} \cdot \frac{1}{2} \left| \begin{vmatrix} -5 & 0 & 0 \\ -8 & 0 & 1 \\ -4 & 2 & 0 \end{vmatrix} \right|$$

$$= \frac{1}{6} \left| -2 \begin{vmatrix} -5 & 0 \\ -8 & 1 \end{vmatrix} \right| = \frac{1}{6} \cdot 2 \cdot |-5| = \underline{\underline{\frac{5}{3}}}$$

17 $\quad \vec{a}, \vec{b}, \vec{c}, \vec{d}$ vektorer

$\vec{a} = (a_1 \; a_2 \; a_3)$

$\vec{b} = (b_1 \; b_2 \; b_3)$

$\vec{c} = (c_1 \; c_2 \; c_3)$

$\vec{d} = (d_1 \; d_2 \; d_3)$

a) $\vec{a} = \vec{b} = \vec{c}$

$\vec{a} = \vec{b}$ eller

$\vec{a} = \vec{c}$ eller $\quad \Rightarrow \quad 0 = V = |\det(\vec{a}, \vec{b}, \vec{c})|$

$\vec{b} = \vec{c}$

$$= \left| \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \right| = 0$$

b)

$$\det(s\vec{a} + t\vec{d}, \vec{b}, \vec{c})$$

$$((s\vec{a} + t\vec{d}) \times \vec{b}) \cdot \vec{c} = (s\vec{a} \times \vec{b}) \cdot \vec{c} + (t\vec{d} \times \vec{b}) \cdot \vec{c}$$

$$= s(\vec{a} \times \vec{b}) \cdot \vec{c} + t(\vec{d} \times \vec{b}) \cdot \vec{c}$$

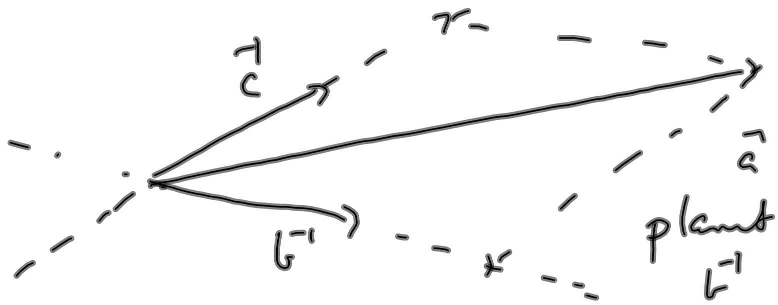
$$= s \det(\vec{a}, \vec{b}, \vec{c}) + t \det(\vec{d}, \vec{b}, \vec{c})$$

c) $\vec{a} = s\vec{b} + t\vec{c} \quad \Rightarrow$

$$\det(\vec{a}, \vec{b}, \vec{c}) = \det(s\vec{b} + t\vec{c}, \vec{b}, \vec{c})$$

$$\overset{(b)}{=} s \det(\vec{b}, \vec{b}, \vec{c}) + t \det(\vec{c}, \vec{b}, \vec{c})$$

$$\overset{(a)}{=} s \cdot 0 + t \cdot 0 = 0.$$

d) 

$\vec{a} = s\vec{b} + t\vec{c}$

$\vec{a}$ ligger i planet utspent av $\vec{b}$ og $\vec{c}$

så volumet utspent av $\vec{a}, \vec{b}, \vec{c}$ er 0

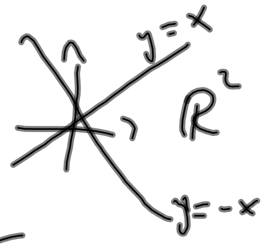
2.1 1

$$f = \frac{1}{x^2+4y^2}$$

D_f

$\mathbb{R}^2 \setminus \{(0,0)\} \subseteq \mathbb{R}^2$

$(x,y) \neq (0,0)$



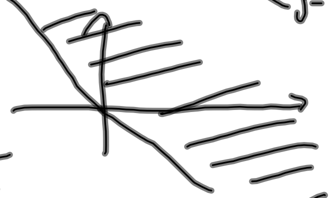
$$f = \frac{1}{x^2-y^2}$$

$y \neq \pm x$

$y \neq x$ og $y \neq -x$

$$f = \ln(x+y)$$

$x+y>0$



$$f = \tan(x-y)$$

$x-y \neq \frac{\pi}{2} + k\pi$

$$f = \frac{1}{x^2+y^2+z^2-25}$$

$x^2+y^2+z^2 \neq 25$

$\{(x,y,z) \mid \|(x,y,z)\| \neq 5\}$

2.2.4 $\quad M \in \mathbb{R}$

$$|\vec{F}(\vec{x}) - \vec{F}(\vec{y})| \leq M\,|\vec{x} - \vec{y}|$$

for alle $\vec{x}, \vec{y}$

Gitt $\varepsilon > 0$

$$|\vec{F}(\vec{x}) - \vec{F}(\vec{y})| \leq M\,|\underbrace{\vec{x} - \vec{y}}_{\delta}|$$

$$< M\delta < \varepsilon$$

når $\delta < \frac{\varepsilon}{M}$ 0

b) $\vec{F}(\vec{x}) = A\vec{x}$ $\qquad$ $\vec{F}(\vec{x}) - \vec{F}(\vec{y})$

matrise

$$= A\vec{x} - A\vec{y}$$

$$= A(\vec{x} - \vec{y})$$

$$\Rightarrow |\vec{F}(\vec{x}) - \vec{F}(\vec{x})| = |A(\vec{x} - \vec{y})|$$

$$\leq \|A\|\,|\vec{x} - \vec{y}| \quad \text{for alle } \vec{x}, \vec{y}.$$

av a) er $\vec{F}$ kontinuerlig.

2.3   1.d   2

$$\lim_{(x,y)\to(0,0)} \frac{\sin xy}{xy} \cos(x+y)$$

$$f_1 = \frac{\sin xy}{xy}$$

$$(x,y) \longmapsto xy \longmapsto \frac{\sin xy}{xy}$$

$$u \longmapsto \frac{\sin u}{u}$$

$$h(x,y) = xy \qquad g(u) = \frac{\sin u}{u}$$

$f_1 = g \circ h$, h kont og g kont i $(0,0)$ og 0.

så $\lim_{x,y\to 0,0} \frac{\sin xy}{xy} = 1$ $\lim_{x,y\to(0,0)} \cos(x+y)$.

$$\Rightarrow \lim_{x,y\to 0,0} f(x,y) = 1 \cdot 1 = 1$$

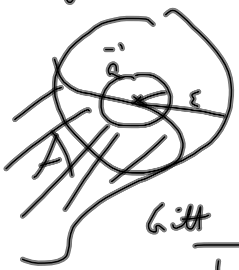
nov 28-13:43

2. $A \subseteq \mathbb{R}^n \qquad \vec{a} \in \mathbb{R}^n$

Anta at $B(\vec{a}, \varepsilon) \cap A \ni \vec{y}_\varepsilon \neq \vec{a}$

for alle ε.

Vis at $\vec{a}$ er et opphopningspunkt for A



det vil si at $B(\vec{a}, \varepsilon) \cap A$ inneholder uendelig mange punkter.

Gitt ε finner $\vec{y}_\varepsilon \in B(\vec{a}, \varepsilon) \cap A$

$\vec{y}_\varepsilon \neq \vec{a}$

$0 < |\vec{y}_\varepsilon - \vec{a}| = \varepsilon_1$

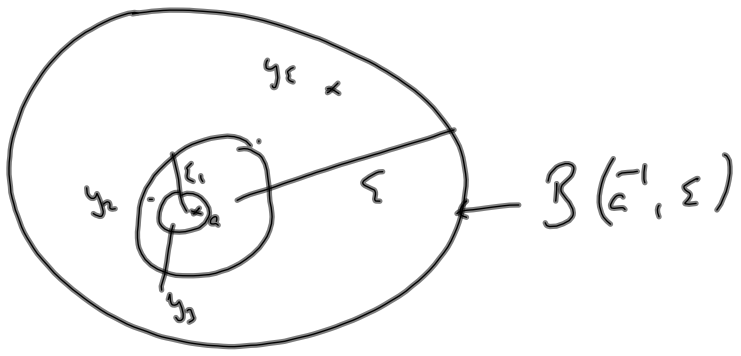
$B(\vec{a}, \frac{\varepsilon_1}{2}) \cap A \ni \vec{y}_2 \neq \vec{a}$

$0 < |\vec{y}_2 - \vec{a}| = \varepsilon_2 \qquad B(\vec{a}, \frac{\varepsilon_2}{2}) \cap A \ni \vec{y}_3 \neq \vec{a}$

osv.

Får da $\vec{y}_\varepsilon, \vec{y}_2, \vec{y}_3, \dots \vec{y}_n$ – –

uendelig mange elementer i A, de ligger alle i $B(\vec{a}, \varepsilon)$!.



Så $\vec{a}$ er opphopningspunkt for A.