8.3 7 ab, 9, 13

7a

$$\lim_{x\to 0} \frac{\int_0^x e^{-t^2}dt}{x} \overset{l'H}{=} \lim_{x\to 0} \frac{e^{-x^2}}{1} = \underline{1}$$

9. f kontinuerlig

$$\int_a^b f(x)\,dx = f(c)(b-a) \quad \text{for en } c \in (a,b)$$

La $K(x) = \int_a^x f(t)\,dt$

Da er $K(x)$ deriverbar (kont.)

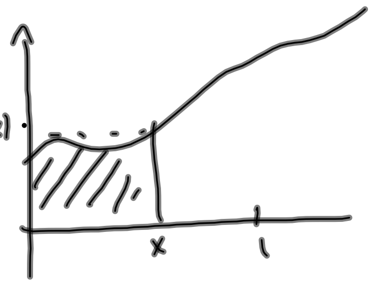
Da er $\dfrac{K(b)-K(a)}{b-a} = K'(c)$ for en $c \in (a,b)$

$$\frac{\int_a^b f(t)\,dt - 0}{b-a} = f(c) \Rightarrow \int_a^b f(x)\,dx = f(c)(b-a)$$

nov 3-12:13

13) g positiv og kontinuerlig på $[0, \infty)$

$$h(x) = \int_0^x g(t)\,dt$$



a) $h'(x) = g(x) > 0 \quad \forall x$

$\Rightarrow$ h strengt voksende

$h(0) = 0$ så $h(x) > 0$ når $x > 0$.

b) $h(c) < h(x) = \int_0^x g(t)\,dt \leq g(x) \qquad x \in [0,1]$

$= g(c) \cdot x \qquad 0 \leq c \leq x$

$\leq g(c) \Rightarrow h(c) < g(c)$

c) $h(x) = \int_0^x g(t)\,dt \leq g(x) \qquad x \in [0,1]$

$a_1(x) = g(x)$

$a_2(x) = \int_0^x a_1(t)\,dt$

$a_3(x) = \int_0^x a_2(t)\,dt$ etc

gitt $x \in [0,1)$ har vi: $a_1(x) \geq a_2(x) \geq a_3(x) \ldots$

og at $a_1(x) \geq 0$, $a_2(x) \geq 0$, $a_3(x) \geq 0$. ...

Så $\{a_n(x)\}_{n=1}^{\infty}$ er en monoton og begrenset følge.

Derfor er den konvergent.

nov 3-12:25

1\. e $\displaystyle\int \frac{4}{\sqrt{7-x^2}}\,dx = 4\cdot\frac{1}{\sqrt{7}}\int \frac{1}{\sqrt{1-\frac{x^2}{7}}}\,dx$

$\left(u = \frac{x}{\sqrt{7}}\right) \qquad \displaystyle = 4\,\frac{1}{\sqrt{7}}\int \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{7}}\right)^2}}\,dx$

$\displaystyle = 4\int \frac{\frac{1}{\sqrt{7}}}{\sqrt{1-\left(\frac{x}{\sqrt{7}}\right)^2}}\,dx = 4\arcsin\left(\frac{x}{\sqrt{7}}\right)$

<u>5</u> $f : (0, \infty) \to \mathbb{R}$

$f(xy) = f(x) + f(y)$ $\qquad$ f er deriverbar i $x=1$

$f'(1) = k$

a) $f(y) = f(1\cdot y) = f(1) + f(y)$

$\Rightarrow f(1) = f(y) - f(y) = 0.$

b) $f(x+h) = f\left(x\left(1+\frac{h}{x}\right)\right) = f(x) + f\left(1+\frac{h}{x}\right)$

$\displaystyle \underline{f'(x)} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(f\left(1+\frac{h}{x}\right)\right)$

$\displaystyle = \lim_{h\to 0} \frac{f\left(1+\frac{h}{x}\right)}{h} = \underbrace{\lim_{h\to 0} \frac{f\left(1+\frac{h}{x}\right) - f(1)}{\frac{h}{x}}}_{f'(1)} \cdot \frac{1}{x} = \underline{\underline{\frac{k}{x}}}$

c) $f'(x) = \frac{k}{x}$

$\displaystyle f(x) = \int \frac{k}{t}\,dt + C = k\ln x + C$

$f(1) = 0 \Rightarrow C = 0$ så $\underline{f(x) = k\ln x}$

8.5 1a

$f(x) = x^2 \qquad \Pi = [\underbrace{1, \tfrac{3}{2}}, \underbrace{\tfrac{3}{2}, 2}, 2, 5/2, 3] \qquad U = \{5/4, 7/4, 9/4, 11/4\}$

$$R(\Pi, U) = \frac{1}{2}\frac{25}{16} + \frac{1}{2}\cdot\frac{49}{16} + \frac{1}{2}\frac{81}{16} + \frac{1}{2}\frac{121}{16} = \_\_$$

2. $f(x) = x \qquad 0, x_1, x_2, \ldots\ x_{n-1}, x_n = a$

$c_i = \frac{x_{i+1} + x_i}{2}$

$$R(\Pi, U) = x_1\left(\frac{x_1}{2}\right) + (x_2 - x_1)\frac{x_2 + x_1}{2} + (x_3 - x_2)\frac{x_3 + x_2}{2}$$

$$+ (x_n - x_{n-1})\frac{x_n + x_{n-1}}{2}$$

$$= \frac{x_n^2}{2} = \frac{a^2}{2} = \int_0^a x\, dx$$

4

$$\lim_{n\to\infty} \frac{1}{n^{3/2}} \sum \sqrt{i} = \lim_{n\to\infty} \frac{1}{n} \sum_{0}^{n} \frac{\sqrt{i}}{\sqrt{n}} = \lim_{n\to\infty} \frac{1}{n}\left(0 + \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$$

$$= \int_0^1 \sqrt{x}\, dx = \underline{\frac{2}{3}}$$

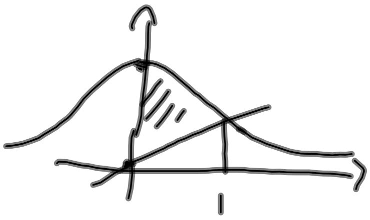
5

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \sum_{1}^{n} \frac{1}{\sqrt{i}} = \lim_{n\to\infty} \frac{1}{n} \sum_{1}^{n} \frac{\sqrt{n}}{\sqrt{i}}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{1}^{n} \frac{1}{\sqrt{\frac{i}{n}}} = \int_0^1 \frac{1}{\sqrt{x}}\, dx = \underline{2}$$

8.6

1.f $\quad y = \frac{1}{1+x^2} \quad y = x/2 \quad x = 0$

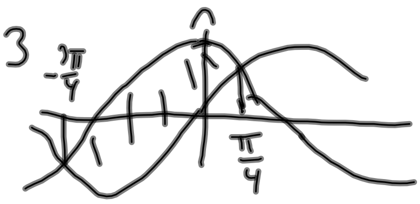


$$\frac{1}{1+x^2} = \frac{x}{2} \Rightarrow \underline{x = 1}$$

$$A = \int_0^1 \left(\frac{1}{1+x^2} - \frac{x}{2} \right) dx$$

$$= \left[\arctan x - \frac{1}{4}x^2 \right]_0^1 = \underline{\underline{\frac{\pi}{4} - \frac{1}{4}}}$$

3

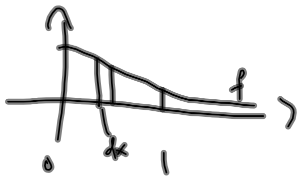


$$\sin x = \cos x \Rightarrow x = \frac{\pi}{4} + k\pi$$

$$A = \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} (\cos x - \sin x)\, dx$$

$$= \left[\sin x + \cos x \right]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2}\sqrt{2} \cdot 2 - \left(-\frac{1}{2}\sqrt{2}\right) \cdot 2 = \underline{2\sqrt{2}}$$

5c

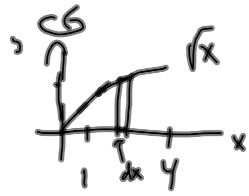


$$dV = \pi \cdot (f(x))^2 \cdot dx$$

$$V = \int dV = \int_0^1 \pi \cdot \left(\frac{1}{\sqrt{1+x^2}}\right)^2 dx = \pi \int_0^1 \frac{1}{1+x^2}\, dx$$

$$= \pi \left[\arctan x\right]_0^1 = \pi \cdot \frac{\pi}{4} = \underline{\underline{\frac{\pi^2}{4}}}$$

7 b)



$$dV = 2\pi x f(x) \cdot dx$$

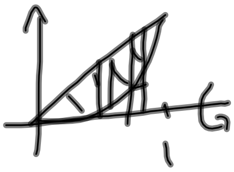
$$V = \int dV = \int_1^4 2\pi x \cdot \sqrt{x}\, dx = 2\pi \int_1^4 x^{3/2} dx$$

$$= 2\pi \left[\frac{2}{5} x^{\frac{5}{2}}\right]_1^4 = 2\pi\left(\frac{64}{5} - \frac{2}{5}\right)$$

$$= \underline{\frac{124\pi}{5}}$$

nov 3-13:24

9.

a) 

$$dV = \pi(x^2 - x^4)\cdot dx$$

$$V = \int dV = \int_0^1 \pi(x^2 - x^4)\,dx = \pi\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1$$

$$= \frac{2}{15}\pi$$

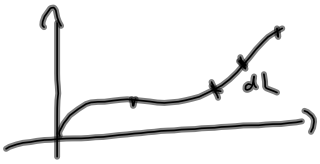
b) 

$$dV = 2\pi x\,(x - x^2)\,dx$$

$$V = \int dV = 2\pi\int x^2 - x^3\,dx = 2\pi\left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1$$

$$= 2\pi\,\frac{1}{12} = \frac{\pi}{6}$$

11 c.



$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx$$

$$L = \int dL$$

$$y = \frac{x^2}{2} - \frac{1}{4} \ln x$$

$$\frac{dy}{dx} = x - \frac{1}{4x}$$

$$= \int_1^e \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\, dx$$

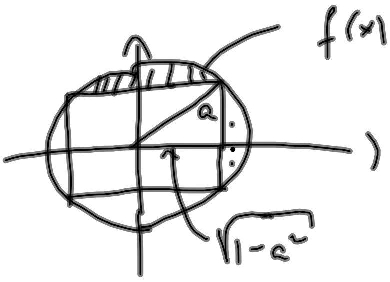
$$= \int_1^e \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{(4x)^2}}\, dx = \int_1^e \sqrt{\underbrace{x^2 + \frac{1}{2} + \frac{1}{(4x)^2}}_{= \left(x + \frac{1}{4x}\right)^2}}\, dx$$

$$= \int_1^e \left(x + \frac{1}{4x}\right) dx = \left[\frac{1}{2} x^2 + \frac{1}{4} \ln x\right]_1^e = \frac{e^2}{2} + \frac{1}{4} - \frac{1}{2}$$

$$= \underline{\frac{e^2}{2} - \frac{1}{2}}$$

15.

$f(x)=\sqrt{1-x^2}$ $\qquad dV=\pi\left(f(x)^2-a^2\right)dx$

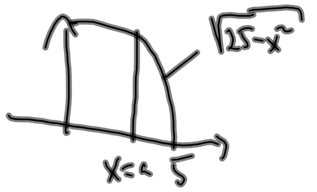


$$V=\int dV$$

$$=2\pi\int_0^{\sqrt{1-a^2}}(1-x^2-a^2)\,dx$$

$$=2\pi\left[(1-a^2)x-\frac{1}{3}x^3\right]_0^{\sqrt{1-a^2}}.$$

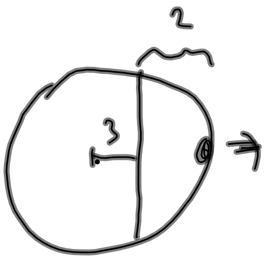
26 $\qquad 0<a<5$



$V=\int dV \qquad dV=\pi(25-x^2)\cdot dx$

$$=\pi\int_0^a(25-x^2)\,dx=\pi\left[25x-\frac{1}{3}x^3\right]_0^a$$

$$=\underline{\pi\left(25a-\frac{a^3}{3}\right)}$$



$$\frac{dV}{dt}=0.5\ m^3/min$$

$$0.5=\frac{dV}{dt}=\frac{dV}{dx}\cdot\frac{dx}{dt}=\pi\cdot(25-x^2)\cdot\frac{dx}{dt}$$

$x=3$

$$=\pi\,16\cdot\frac{dx}{dt}$$

$$\frac{dx}{dt}=\frac{0.5}{\pi\cdot16}\sim\underline{0.01}$$

9.1. 1f

$$\int \arcsin x \, dx = x \cdot \arcsin x - \int x \frac{1}{\sqrt{1-x^2}} \, dx$$

$$= x \arcsin x + \frac{1}{2} \sqrt{1-x^2} \quad . + C$$

5. $$\int \frac{\ln(x^2)}{x^2} = \left(-\frac{1}{x}\right) \cdot \ln(x^2) - \int \left(-\frac{1}{x}\right) \frac{1}{x^2} \cdot 2x \, dx$$

$$= \underline{-\frac{\ln x^2}{x} - 2 \cdot \frac{1}{x}} + C$$

9. $$\int \underline{\sin \ln(x) \, dx} = x \sin(\ln x) - \int x \cos \ln x \cdot \frac{1}{x} \, dx$$

$$= x \sin \ln x - \int \cos \ln x \, dx$$

$$= x \sin \ln x - x \cos \ln x + \int \sin \ln x \cdot \frac{1}{x} \cdot x \, dx$$

$$= x \sin \ln x - x \cos \ln x - \underline{\int \sin \ln x \, dx}$$

$$\Rightarrow \int \sin \ln(x) \, dx = \frac{1}{2} \left(x \sin \ln x - x \cos \ln x \right) + C$$

11. $$\int \frac{x^2 \arctan x}{1+x^2}\,dx = \int \frac{(1+x^2)\arctan x}{1+x^2} \quad \frac{-\arctan x}{}\,dx$$

$$= \int \arctan x - \frac{\arctan x}{1+x^2}\,dx$$

$$= x\arctan x - \int \frac{x}{1+x^2}\,dx - \frac{1}{2}(\arctan x)^2$$

$$= x\arctan x - \frac{1}{2}\ln(1+x^2) - \frac{1}{2}(\arctan x)^2 + C$$

nov 3-14:01