9.2 1 b d g h, 3c, 7, 9, 15, 23, 25

1g $\int \cos(\ln x)\,dx = \int \cos u \cdot e^{-u}\,du$

$u = \ln x \quad x = e^u$

$\frac{du}{dx} = \frac{1}{x} \quad du = \frac{1}{x}dx = e^u dx \quad dx = e^{-u}du$

$= -e^{-u}\cos u - \int -e^{-u}(-\sin u)\,du = -e^{-u}\cos u - \int e^{-u}\sin u\,du$

$= -e^{-u}\cos u - (-e^{-u})\sin u + \int (-e^{-u})\cos u\,du$

$= -e^{-u}\cos u + e^{-u}\sin u - \int e^{-u}\cos u\,du$

$\int e^{-u}\cos u\,du = \frac{1}{2}\left(-e^{-u}\cos u + e^{-u}\sin u\right) + C$

$\int \cos(\ln x)\,dx = \frac{1}{2}\left(-e^{-\ln x}\cos(\ln x) + e^{-\ln x}\sin(\ln x)\right) + C$

$= \frac{1}{2}\left(-\frac{1}{x}\cos(\ln x) + \frac{1}{x}\sin(\ln x)\right) + C$

1.h $\int \arcsin\sqrt{x}\,dx = \int \arcsin u \cdot 2u\,du$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2u}\,dx \Rightarrow dx = 2u\,du$

$= u^2 \arcsin u - \int u^2 \cdot \frac{1}{\sqrt{1-u^2}}\,du$

$= u^2 \arcsin u + \int \frac{-u^2+1-1}{\sqrt{1-u^2}}\,du$

$= u^2 \arcsin u + \int \frac{1-u^2}{\sqrt{1-u^2}}\,du - \int \frac{+1}{\sqrt{1-u^2}}\,du$

$= u^2 \arcsin u - \arcsin u + \int \sqrt{1-u^2}\,du$

$u = \sin v \quad du = \cos v\,dv$

$= u^2 \arcsin u - \arcsin u + \int \cos^2 v\,dv$

$\int \cos^2 v\,dv = \int \frac{\cos 2v + 1}{2}\,dv = \frac{1}{2\cdot 2}\sin 2v + \frac{v}{2} + C$

$\int \arcsin\sqrt{x}\,dx = x \arcsin\sqrt{x} - \arcsin\sqrt{x}$

$+ \frac{1}{4}\sin\left(2\arcsin\sqrt{x}\right) + \frac{1}{2}\arcsin\sqrt{x} + C$

3c $\displaystyle\int_4^9 \frac{\sqrt{x}+1}{1-\sqrt{x}}\,dx = \int_2^3 \frac{1+u}{1-u}\cdot 2u\,du$

$u = \sqrt{x} \qquad dx = 2u\,du$

$\dfrac{2u+2u^2}{1-u} \qquad (2u^2+2u) : (u-1) = 2u+4$

$\qquad\qquad\qquad \underline{2u^2-2u}$

$\qquad\qquad\qquad\quad 4u$

$\qquad\qquad\qquad\quad \underline{4u-4}$

$\qquad\qquad\qquad\qquad 4$

$= -2u - 4 - \dfrac{4}{1-u}$

$$= \int\left(-2u - 4 + \frac{4}{u-1}\right)du = -u^2 - 4u + 4\ln|u-1| + C$$

$$= -x - 4\sqrt{x} + 4\ln|\sqrt{x}-1| + C$$

$$\left[-x - 4\sqrt{x} + 4\ln|\sqrt{x}-1|\right]_4^9 = -21 + 4\ln 2 + 12 + 0$$

$$= 4\ln 2 - 9$$

7 $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx = \int \frac{\sqrt{1+u}}{u} \cdot 2u \, du$

$u = \sqrt{x} \quad dx = 2u \, du$

$= 2\int (1+u)^{\frac{1}{2}} \, du = 2 \, \frac{2}{3}(1+u)^{\frac{3}{2}} + C = \frac{4}{3} \left(1+\sqrt{x}\right)^{\frac{3}{2}} + C$

15

$$\int_0^{\sqrt{3}} \frac{1+x}{\sqrt{4-x^2}}\,dx = \int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx + \int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx$$

$$= \int \frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx + \left(-\frac{1}{2}\right)\int \frac{-2x}{\sqrt{4-x^2}}\,dx$$

$u = 4-x^2$

$$= \left[\frac{1}{2}\,2\,\arcsin\left(\frac{x}{2}\right) - \frac{1}{2}\,2\cdot\left(4-x^2\right)^{\frac{1}{2}}\right]_0^{\sqrt{3}} + \cancel{C}$$

$$= \left[\arcsin\frac{x}{2} - \sqrt{4-x^2}\right]_0^{\sqrt{3}} + \cancel{C}$$

$$= \frac{\pi}{3} - 1 + 2 = \underline{\frac{\pi}{3} + 1}$$

25 $\quad I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx \qquad n = 0, 1, 2, \dots$

a) $I_0 = \int_0^{\frac{\pi}{4}} 1 \, dx = \frac{\pi}{4}$

$I_1 = \int_0^{\frac{\pi}{4}} \tan x \, dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$= -\int_1^{\frac{1}{\sqrt{2}}} \frac{du}{u} = -\left[\ln u\right]_1^{\frac{1}{\sqrt{2}}} = \underline{\frac{1}{2}\ln 2}$

b) $\tan^{n+2} x = \tan^n x \left(\tan^2 x\right) = \tan^n x \left(\frac{\sin^2 x + \cos^2 x - \cos^2 x}{\cos^2 x}\right)$

$= \tan^n x \left(\frac{1}{\cos^2 x} - 1\right)$

$I_{n+2} = \int_0^{\frac{\pi}{4}} \tan^{n+2} x = \int_0^{\frac{\pi}{4}} \tan^n x \cdot \frac{1}{\cos^2 x} \, dx - \int_0^{\frac{\pi}{4}} \tan^n x \, dx$

$u = \tan x \quad du = \frac{1}{\cos^2 x} dx$

$= \int_0^1 u^n \, du - I_n = \underline{\frac{1}{n+1} - I_n}$

c)(*) $I_{2n+1} = \frac{(-1)^n}{2}\left[\ln 2 - \left(1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^{n+1}}{n}\right)\right]$

$\underline{n=1}$ $\underline{VS}$: $I_3 = \frac{1}{2} - I_1 = \frac{1}{2} - \frac{1}{2}\ln 2$

$\underline{HS}$: $\frac{-1}{2}\left[\ln 2 - 1\right] \qquad VS = HS \;\checkmark$

Anta (*) er ok for $n-1$:

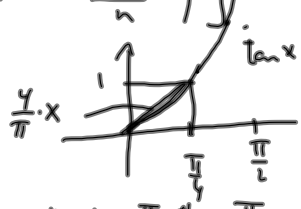
$\underline{\text{Av b}}$: $I_{2n+1} = \frac{1}{2n} - I_{2n-1}$

$= \frac{1}{2n} - \frac{(-1)^{n-1}}{2}\left[\ln 2 - \left(1 - \frac{1}{2} + \dots + \frac{(-1)^n}{n-1}\right)\right]$

$= \frac{(-1)^n}{2}\left[\ln 2 - \left(1 - \frac{1}{2} + \dots + \frac{(-1)^n}{n-1}\right) + \frac{(-1)^n}{n}\right]$

$= \frac{(-1)^n}{2}\left[\ln 2 - \left(1 - \frac{1}{2} + \dots + \frac{(-1)^{n+1}}{n}\right)\right]$

d) $\lim_{n\to\infty} \int_0^{\frac{\pi}{4}} \tan^n x \, dx$



$0 \le \int_0^{\frac{\pi}{4}} \tan^n x \, dx \le \int_0^{\frac{\pi}{4}} \left(\frac{4}{\pi} x\right)^n dx$

$= \left(\frac{4}{\pi}\right)^n \left[\frac{1}{n+1} x^{n+1}\right]_0^{\frac{\pi}{4}} = \left(\frac{4}{\pi}\right)^n \cdot \frac{1}{n+1}\left(\frac{\pi}{4}\right)^{n+1} = \frac{\pi}{4 \cdot (n+1)} \xrightarrow[n\to\infty]{} 0$

så $\lim_{n\to\infty} \int_0^{\frac{\pi}{4}} \tan^n x \, dx = 0$

$\Rightarrow \lim_{n\to\infty} \left[\ln 2 - \left(1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^{n+1}}{n}\right)\right] = 0$

$\Rightarrow \ln 2 = \lim_{n\to\infty} \left(1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{(-1)^{n+1}}{n}\right)$

9.3 1. d 5 f 9 21 25 27

1.d
$$\int \frac{x+7}{x^2-x-2}\,dx = \int \frac{3}{x-2} - \frac{2}{x+1}\,dx = \underline{3\ln|x-2| - 2\ln|x+1| + C}$$

$$x^2 - x - 2 = (x-2)(x+1)$$

$$\frac{x+7}{x^2-x-2} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{Ax + A + Bx - 2B}{(x-2)(x+1)}$$

$$\Rightarrow \quad A + B = 1 \quad A - 2B = 7 \quad \to \quad B - (-2B) = 3B = -6$$

$$B = -2 \quad A = 3$$

nov 10-13:27

$\underline{21}$ a) $\displaystyle\int \frac{u+2}{u^2+2u+5}\,du = \int \frac{u+2}{(u+1)^2+4}\,du = \frac{1}{2}\int\frac{2(u+1)}{(u+1)^2+4}\,du + \int\frac{1}{(u+1)^2+4}\,du$

$\displaystyle = \frac{1}{2}\ln\left((u+1)^2+4\right) + \frac{1}{4}\,2\cdot\arctan\left(\frac{u+1}{2}\right) + C$ $\qquad \displaystyle \text{"} \;\; \frac{\frac{1}{4}}{\left(\frac{u+1}{2}\right)^2+1}$

$\displaystyle = \frac{1}{2}\ln(u^2+2u+5) + \frac{1}{2}\arctan\frac{u+1}{2} + C$

b) $\displaystyle \frac{1}{u(u^2+2u+5)} = \frac{A}{u} + \frac{Bu+C}{u^2+2u+5} = \frac{Au^2+2Au+5A+Bu^2+Cu}{u(u^2+2u+5)}$

$\Rightarrow \begin{aligned} A+B &= 0 \\ 2A+C &= 0 \\ 5A &= 1 \end{aligned} \qquad \underline{A=\frac{1}{5}} \quad \underline{B=-\frac{1}{5}} \quad \underline{C=-\frac{2}{5}}$

c) $\displaystyle\int \frac{\sin x}{\cos^2 x + 2\cos x + 5}\,dx = \int\frac{-\,du}{u(u^2+2u+5)} = -\int\frac{du}{u(u^2+2u+5)}$

$u = \cos x \qquad du = -\sin x\,dx$

$\displaystyle = -\frac{1}{5}\int\frac{du}{u} + \frac{1}{5}\int\frac{u+2}{u^2+2u+5}\,du$

$\displaystyle = -\frac{1}{5}\ln|u| + \frac{1}{5}\left(\frac{1}{2}\ln(u^2+2u+5) + \frac{1}{2}\arctan\frac{u+1}{2}\right)$

$\displaystyle = -\frac{1}{5}\ln|\cos x| + \frac{1}{10}\ln(\cos^2 x + 2\cos x + 5) + \frac{1}{10}\arctan\left(\frac{\cos x+1}{2}\right) + C$

25 $\quad z^3 - 11z + 20 \qquad z_0 = 2+i$

$z = z_0: \quad (2+i)^3 - 11(2+i) + 20$

$= 8 + 3\cdot 4\cdot i + 3\cdot 2\cdot(-1) - i - 22 - 11i + 20 = \underline{0}$

z_0 er rot i ligningen.
Siden koeffisientene er reelle så er også $\bar{z}_0 = 2-i$ en rot.

$(z-2-i)(z-2+i) = z^2 - 4z + 4 + 1$

$= \underline{z^2 - 4z + 5}$

$$\begin{array}{l} (z^3 - 11z + 20) : (z^2 - 4z + 5) = \underline{z+4} \\ \underline{z^3 - 4z^2 + 5z} \\ \quad 4z^2 - 16z + 20 \\ \quad \underline{4z^2 - 16z + 20} \\ \qquad\qquad\quad 0 \end{array}$$

$\underline{z = -4}$ er $\underline{\text{rot}}$

$z^3 - 11z + 20 = (z^2 - 4z + 5)(z+4)$.

4 $\displaystyle\int \frac{10x+3}{x^3 - 11x + 20}\,dx = \int \frac{x+2}{x^2-4x+5}\,dx + \int \frac{-dx}{x+4} \quad \ldots$

$$\frac{10x+3}{x^3 - 11x + 20} = \frac{Ax+B}{x^2-4x+5} + \frac{C}{x+4} = \frac{Ax^2 + 4Ax + Bx + 4B + Cx^2 - 4Cx + 5C}{(x+4)(x^2-4x+5)}$$

$$\Rightarrow \begin{array}{l} A + C = 0 \\ 4A + B - 4C = 10 \\ 4B + 5C = 3 \end{array} \qquad A = -C \qquad \Rightarrow \left.\begin{array}{l} B - 8C = 10 \\ 4B + 5C = 3 \end{array}\right\} \quad \begin{array}{l} 37C = -37 \\ C = -1 \end{array}$$

$B = 2 \qquad A = 1$

27. $\int \frac{dx}{e^{2x}+4e^x+13} = \int \frac{\frac{1}{u}\,du}{u^2+4u+13} = \int \frac{du}{u(u^2+4u+13)}$

$u = e^x \quad du = e^x\,dx = u\,dx \rightarrow dx = \frac{du}{u}$

$u^2+4u+13 = (u+2)^2+9$

$4+9$

$\frac{1}{u(u^2+4u+13)} = \frac{A}{u} + \frac{Bu+C}{u^2+4u+13}$

$= \frac{Au^2+4Au+13A+Bu^2+Cu}{u(u^2+4u+13)}$

$A+B = 0$

$4A+C = 0$

$13A = 1 \qquad A = \frac{1}{13} \quad B = -\frac{1}{13} \quad C = -\frac{4}{13}$

$= \frac{1}{13}\int \frac{du}{u} - \frac{1}{13}\int \frac{u+4}{u^2+4u+13}\,du$

$= \frac{1}{13}\ln|u| - \frac{1}{13}\cdot\frac{1}{2}\ln|u^2+4u+13| - \frac{1}{13}\int\frac{2}{(u+2)^2+9}\,du$

$\parallel$

$\frac{1}{13}\frac{2}{3}3\arctan\left(\frac{u+2}{3}\right)$