9.4 $\quad \underline{8} \quad 10$

9.5 $\quad$ 1ab $\quad$ 3ac $\quad \underline{6} \quad 10 \quad \underline{12}$

9.4 $\quad \underline{8}$

$$\int \sin^3 x \cos^2 x \; dx \qquad u = \sin^2 x \qquad v' = \sin x \cos^2 x$$

$$= -\frac{1}{3}\sin^2 x \cos^3 x + \frac{1}{3}\int 2\sin x \underline{\cos x} \underline{\cos^3 x} \; dx \qquad u = \cos x$$

$$= -\frac{1}{3}\sin^2 x \cos^3 x + \frac{2}{3}\frac{1}{5}(-1)\cos^5 x + C$$

$$= -\frac{1}{3}\sin^2 x \cos^3 x - \frac{2}{15}\cos^5 x + C$$

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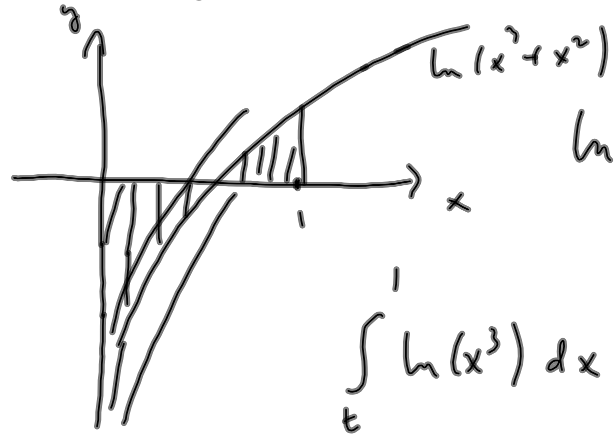
9.5 $\underline{b}$ $\int_0^1 \ln(x^3+x^2)\,dx = \lim_{t\to 0} \int_t^1 \ln(x^3+x^2)\,dx$



$\ln(x^3) < \ln(x^3+x^2) < \ln(2x^2)$

$\underline{x < \tfrac{1}{2}}$

$$\int_t^1 \ln(x^3)\,dx = 3\,(x\ln x - x)\Big]_t^1$$

$$= -3 - t\ln t + t$$

$$\lim_{t\to 0} t\ln t = \lim_{t\to 0} \frac{\ln t}{\frac{1}{t}} \overset{L'H}{=} \lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}}$$

$$= -\lim_{t\to 0} t = 0$$

$$\lim_{t\to 0} \int_t^1 \ln(x^3)\,dx = -3$$

siden $\ln(x^3) < \ln(x^3+x^2) < 0$ når $x < \tfrac{1}{2}$

så får vi at

$\int_0^1 \ln(x^3+x^2)\,dx$ konvergerer.

12 $\int^{\infty} \left(\frac{x}{2x^2+2k} - \frac{k}{x+1} \right) dx$

$$\int \frac{x}{2x^2+2k} - \frac{k}{x+1}\, dx = \int \frac{1}{4}\frac{4x}{2x^2+2k} - \frac{k \cdot 1}{x+1}\, dx$$

$$= \frac{1}{4}\ln\left(2x^2+2k\right) - k\ln(x+1) + C$$

$$= \ln \frac{\left(2x^2+2k\right)^{\frac{1}{4}}}{(x+1)^k} + C$$

$$= \lim_{t\to\infty} \left(\ln \frac{\left(2t^2+2k\right)^{\frac{1}{4}}}{(t+1)^k} - \ln \frac{(2+2k)^{\frac{1}{4}}}{2^k} \right)$$

$$\frac{\left(2t^2+2k\right)^{\frac{1}{4}}}{(t+1)^k} = \frac{\left(2t^2+2k\right)^{\frac{1}{4}}}{(t+1)^{2\cdot\frac{k}{2}}} \quad \begin{array}{l} \to \infty \quad k < \frac{1}{2} \\ \to 0 \quad k > \frac{1}{2} \end{array}$$

når $k = \frac{1}{2}$ er grensen $\lim_{t\to\infty}\left(\frac{2t^2+2\cdot\frac{1}{2}}{(t+1)^2}\right)^{\frac{1}{4}} = 2^{\frac{1}{4}}$

så jeg får konvergens når $k = \frac{1}{2}$ og integralet er da $= \ln 2^{\frac{1}{4}} - \ln \frac{3^{\frac{1}{4}}}{2^{\frac{1}{2}}}$

$$= \frac{1}{4}\ln 2 - \frac{1}{4}\ln 3 + \frac{1}{2}\ln 2$$

$$= \underline{\frac{3}{4}\ln 2 - \frac{1}{4}\ln 3}$$

1.1 $1, 2, 3, 4, 5$

1. $\vec{a} = (1, -2, 4, -5, 1)$ $\vec{b} = (-3, 5, 5, 0, -3)$

$s = 3$

$\vec{a} + \vec{b} = (-2, 3, 9, -5, -2)$

$\vec{a} \cdot \vec{b} = 1 \cdot (-3) + (-2) \cdot 5 + 4 \cdot 5 + (-5) \cdot 0 + 1 \cdot (-3)$

$= -3 - 10 + 20 - 3 = \underline{4}$

5 $m_1, m_2, m_3, \ldots m_n$

$p_1, p_2, p_3, \ldots, p_n$

hel verdi: $m_1 \cdot p_1 + m_2 p_2 + m_3 \cdot p_3 + \quad + m_n \cdot p_n$

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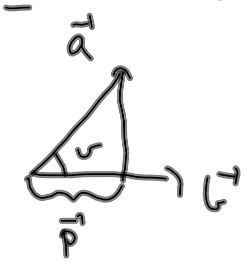
1.2 1, 3, 5, 7, 11, 13, 15, 17, 19, 21, 25, 27

5 $\vec{a} = (4, 3, 1, 2)$ $\quad \vec{b} = (-1, 3, 2, 0)$



$$\cos v = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

$$= \frac{-4 + 9 + 2 + 0}{\sqrt{4^2 + 3^2 + 1^2 + 2^2}\ \sqrt{(-1)^2 + 3^2 + 2^2}}$$

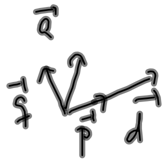
$$= \frac{7}{\sqrt{30} \cdot \sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{15}}$$

$$v = \arccos \frac{\sqrt{7}}{2\sqrt{15}}$$

$$\vec{p} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \cdot \vec{b} = \frac{7}{14}(-1, 3, 2, 0) = \frac{1}{2}(-1, 3, 2, 0)$$

1.2 7 11 13 15 19 20 21 27

7 $\vec{a} = (4,3)$ $\qquad \vec{d} = (1,2)$



$$\vec{a} = \vec{p} + \vec{q} \qquad \vec{p} \cdot \vec{q} = 0$$

$\vec{p} = \alpha \cdot \vec{d}$ for passe α.

$$\vec{p} = \frac{\vec{a} \cdot \vec{d}}{|\vec{d}|^2} \cdot \vec{d} = \frac{4+10}{1+2^2} \cdot \vec{d} = 2(1,2) = (2,4)$$

$$\vec{q} = \vec{a} - \vec{p} = (4,3) - (2,4) = (2,-1)$$

$$\vec{a} = (4,3) = (2,4) + (2,-1)$$

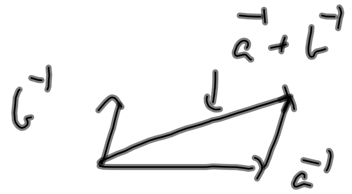
11 $\vec{a} \perp \vec{b}$ og $\vec{a} \perp \vec{c}$

$\Leftrightarrow \vec{a} \cdot \vec{b} = 0$ og $\vec{a} \cdot \vec{c} = 0$

$\vec{a}(\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$

$\Rightarrow \underline{\vec{a} \perp \vec{b} + \vec{c}}$

13 $|\vec{a}| = 3 \quad |\vec{b}| = 2 \quad |\vec{a} + \vec{b}| = 7$



umulig siden

$3 + 2 < 7$

trekantulikheten.

15 $|\vec{x}| - |\vec{y}| \overset{?}{\leq} |\vec{x}-\vec{y}| \qquad \vec{x}, \vec{y} \in \mathbb{R}^n$

trekantulikheten:

$$|\vec{x}| = |\vec{x}-\vec{y} + \vec{y}| \leq |\vec{x}-\vec{y}| + |\vec{y}|$$

så $|x| - |y| \leq |x-y|$

tilsvarende: $|y| - |x| \leq |y-x| = |x-y|$

$$\Rightarrow \big| |x| - |y| \big| \leq |x-y|$$

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19 $\quad (-3, -2, 5, 8) \qquad (1, -2, -1, 3)$

Linje: $(-3, -2, 5, 8) + t(1, -2, -1, 3) \qquad P: (1, -6, 3, 14)$

$$= (-3+t, -2-2t, 5-t, 8+3t) = v(t)$$

P ligger på linja dersom det fins en t slik at $v(t) = P$.

må ha $-3 + t = 1$ d.v.s. $t = 4$

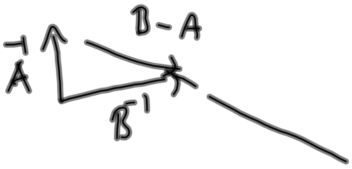
men $-2 - 2\cdot 4 = -10 \neq -6$ så dette går ikke. P er ikke på linja.

20. $\vec{A}: (2,-1,3)$ $\quad \vec{B}: (3,8,-2)$

Linji gjennom A og B :

$$\vec{A} + t(\vec{B} - \vec{A})$$

$$= (2,-1,3) + t(1,9,-5)$$



$$= \underline{(2+t,\ -1+9t,\ 3-5t)}$$

21

$$\vec{A} = (7, -3, 2, 4, -2)$$
$$\vec{B} = (2, 1, -1, -1, 5)$$

$$\vec{A} + t(\vec{B} - \vec{A})$$
$$= (7, -3, 2, 4, -2) + t(-5, 4, -3, -5, 7)$$
$$= (7-5t, -3+4t, 2-3t, 4-5t, -2+7t)$$

27

$|\vec{v}| = 1$ $\qquad \vec{v} = (-\cos u, \sin u)$

a) $(0,6) + t\frac{1}{5}(4,3)\cdot 5$ $\qquad 5\ \text{m s}^{-1}$

$= (0,6) + (4t, 3t) = (4t, 6+3t)$

b) $(20,0) + (t-2)70(-\cos u, \sin u)$ $\qquad 70\text{ms}^{-1}$

$= (20 + (70t-140)(-\cos u), (70t-140)\sin u)$

$= (20 - (70t-140)\cos u, (70t-140)\sin u)$

c) kule treffer desom det fins en t slik at $\quad 20 - (70t-140)\cos u = 4t$

og $\quad (70t-140)\sin u = 6+3t$

$70t\cdot\sin u - 3t = 140\sin u + 6$

$t = \dfrac{140\sin u + 6}{70\sin u - 3}$

$20 - \left(70\cdot\dfrac{140\sin u + 6}{70\sin u - 3} - 140\right)\cos u = 4\cdot\dfrac{140\sin u + 6}{70\sin u - 3}$

$20(70\sin u - 3) - \left(70\cdot(140\sin u + 6) - 140(70\sin u - 3)\right)\cdot\cos u$
$= 4(140\sin u + 6)$

$1400\sin u - 60 - (70\cdot 6 + 140\cdot 3)\cdot\cos u$
$= 4(140\sin u + 6)$

$6\cdot 140\cdot\sin u - 6\cdot 140\cos u = 60 + 24$

$60\sin u - 60\cos u = 6$

$\sin u - \cos u = \frac{1}{10}$

$\frac{1}{\sqrt{2}}\sin u - \frac{1}{\sqrt{2}}\cos u = \frac{1}{10}\cdot\frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\qquad \cancel{\frac{2}{2\sqrt{5}}\sin u + \frac{2}{\sqrt{5}}\cos u = -\frac{1}{10}\cdot\frac{2}{\sqrt{5}}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\qquad \cancel{\frac{1}{\sqrt{5}}\sin u + \frac{2}{\sqrt{5}}\cos u = -\frac{1}{10} - \frac{2}{\sqrt{5}}}$

$\sin(u - \frac{\pi}{4}) = \frac{1}{10\cdot\sqrt{2}}$

$u - \frac{\pi}{4} = \arcsin\left(\frac{1}{10\cdot\sqrt{2}}\right)$

$u = \frac{\pi}{4} + \arcsin\frac{1}{10\cdot\sqrt{2}}$

$u \approx 49°$ $\qquad \sin u \sim 0.71$

$t_0 = \dfrac{140\sin u + 6}{70\sin u - 3} \sim \dfrac{106}{46} \sim 2.1$

høyde : $6 + 3\cdot t_0 = 6 + 3\cdot 2.1 \approx \underline{12.3}$.

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9.3 3a

$$\int \frac{2}{x^2+6x+10}\,dx = \int \frac{2}{(x+3)^2+1}\,dx$$

$$= 2\int \frac{1}{(x+3)^2+1}\,dx = 2\arctan(x+3) + C$$

b) $$\int \frac{2x-2}{x^2+4x+8}\,dx = \int \frac{2x+4}{x^2+4x+8} - \frac{6}{x^2+4x+8}\,dx$$

$$= \ln|x^2+4x+8| - 6\int \frac{1}{(x+2)^2+4}\,dx$$

$$= \ln|x^2+4x+8| - 6\int \frac{\frac{1}{4}}{\left(\frac{x+2}{2}\right)^2+1}\,dx$$

$$= \ln|x^2+4x+8| - \frac{6}{4}\cdot 2\arctan\left(\frac{x+2}{2}\right) + C$$

$$= \ln|x^2+4x+8| - 3\arctan\left(\frac{x+2}{2}\right) + C$$

e) $$\int \frac{x+4}{x^2+4x+3}\,dx =$$

$$\frac{x+4}{x^2+4x+3} = \frac{A}{x+1} + \frac{B}{x+3} \Rightarrow A+B=1 \quad 3A+B=4$$

$$\Rightarrow 2A = 3 \quad A = \frac{3}{2}$$

$$B = -\frac{1}{2}$$

$$\int \frac{x+4}{x^2+4x+3}\,dx = \frac{3}{2}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{dx}{x+3}$$

$$= \frac{3}{2}\ln|x+1| - \frac{1}{2}\ln|x+3| + C$$

5f $\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx$

$$\frac{3x^2+x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\Rightarrow A(x^2+2x+1) + B(x^2-1) + C(x-1) = 3x^2+x$$

$A+B = 3 \quad 2A + C = 1 \quad A - B - C = 0$

$\Rightarrow 2A - C = 3 \quad \Rightarrow A = 1,\ C = -1$

$B = 2$

$$\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx = \int \frac{dx}{x-1} + \int \frac{2\,dx}{x+1} - \int \frac{dx}{(x+1)^2}$$

$$= \ln|x-1| + 2\ln|x+1| - \frac{1}{x+1} + C$$

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9.4.

10 $I = \int \sin^4 x \cos^2 x \, dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$I = \frac{1}{8}\int (1-\cos 2x)^2 (1+\cos 2x) \, dx$

$= \frac{1}{8}\int (1-\cos^2 2x)(1-\cos 2x) \, dx$

$= \frac{1}{8}\int 1 - \cos 2x - \cos^2 2x + \cos^3 2x \, dx$

$= \frac{1}{8}\int 1 - \cos 2x - \frac{1}{2} - \frac{1}{2}\cos 4x + \cos 2x - \cos 2x \sin^2 2x \, dx$

$= \frac{1}{8}\left(x - \frac{1}{2}\sin 2x - \frac{x}{2} - \frac{1}{8}\sin 4x + \frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x\right) + C$

$= \frac{x}{16} - \frac{1}{64}\sin 4x - \frac{1}{48}\sin^3 2x + C$

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