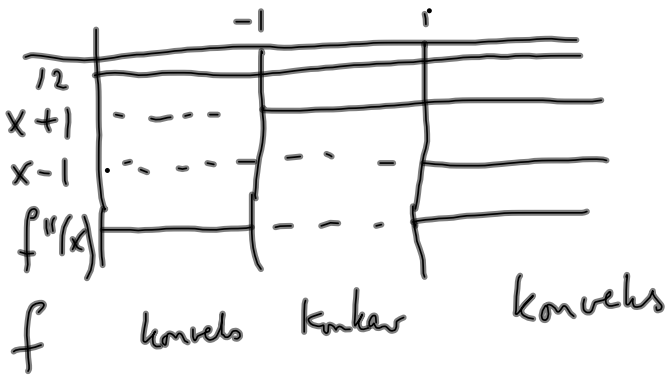
6.4 3a , 7.1 1, 5, 7, 8, 15

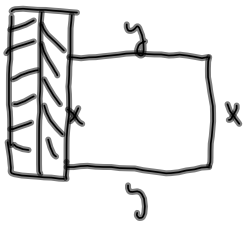
3a $f(x) = x^4 - 6x^2 + 23$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12 = 12(x+1)(x-1)$



7.1 1.



$2y + x = 50$

$A(x) = x \cdot y = x\left(\frac{50 - x}{2}\right)$

$\frac{d}{dx} A \quad \frac{d}{dx}\left(25x - \frac{1}{2}x^2\right) = 25 - x = 0$

hai $x = 25$

$A_{max} = A(25) = 25 \cdot \frac{25}{2} = \underline{312.5}$

5

$V = \frac{1}{3}\pi r^2 \cdot h$

$r^2 = L^2 - h^2$ $\qquad V(h) = \frac{1}{3}\pi (L^2 - h^2) h$

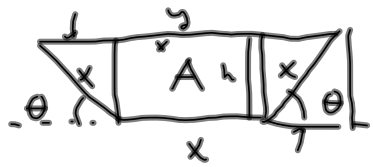
$h = \sqrt{L^2 - r^2}$ $\qquad V(r) = \frac{1}{3}\pi r^2 \sqrt{L^2 - r^2}$

$\frac{d}{dh} V(h) = \frac{1}{3}\pi (L^2 - 3h^2) = 0$

$h^2 = \frac{1}{3} L^2 \qquad h = L \cdot \frac{1}{\sqrt{3}}$

$V_{max} = \frac{1}{3}\pi \left(L^2 - \frac{1}{3}L^2\right)\frac{L}{\sqrt{3}} = \frac{2\pi}{9} \cdot L^2 \frac{L}{\sqrt{3}} = \underline{18\pi \cdot \frac{9}{\sqrt{3}}}$

$= \underline{54\pi\sqrt{3}}$

7. 

$$A = \frac{x+y}{2} \cdot h$$

$$h = x \sin\theta$$

$$y = 2x\cos\theta + x$$

$$A(\theta) = (x + x\cos\theta)\, x \sin\theta = x^2(\sin\theta + \sin\theta\cos\theta)$$

$3x = 60$

$x = 20$

$$= 400(\sin\theta + \sin\theta\cos\theta)$$

$$A'(\theta) = 400(\cos\theta + \cos^2\theta - \sin^2\theta) = 0$$

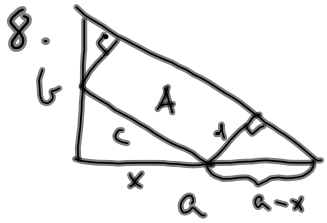
nàn $\cos\theta + 2\cos^2\theta - 1 = 0$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$\cos\theta = \frac{-1 \pm \sqrt{1+8}}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$$

A_{max} nàn $\cos\theta = \frac{1}{2}$, dus wil sê $\theta = \frac{\pi}{3}$

8.



$$A = c \cdot d$$

$$\frac{c}{x} = \frac{\sqrt{a^2+b^2}}{a} \Rightarrow c = \frac{x}{a}\sqrt{a^2+b^2}$$

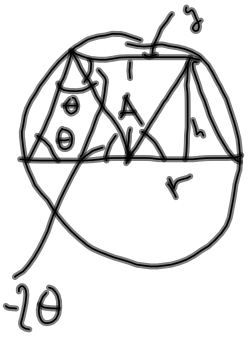
$$\frac{d}{a-x} = \frac{b}{\sqrt{a^2+b^2}} \Rightarrow d = (a-x)b \cdot \frac{1}{\sqrt{a^2+b^2}}$$

$$A(x) = \frac{x}{a} \cdot (a-x) \cdot b = \frac{b}{a}(ax - x^2)$$

$$A'(x) = \frac{b}{a}(a - 2x) = 0 \qquad x = \frac{a}{2}.$$

A_{max} när $x = \frac{a}{2}$.

15.



trapes i en sirkel

$$A = \frac{2r + y}{2} \cdot h$$

$$h = r \sin(\pi - 2\theta) = r \sin 2\theta$$

$$\frac{y}{2} = r \cos(\pi - 2\theta) = -r \cos 2\theta$$

$$A(\theta) = (r + (-r \cos 2\theta)) r \sin 2\theta$$

$$= r^2 (\sin 2\theta - \sin 2\theta \cos 2\theta)$$

$$A'(\theta) = r^2 (2 \cos 2\theta - (2\cos^2 2\theta - 2\sin^2 2\theta))$$

$$= 2r^2 (-2\cos^2 2\theta + \cos 2\theta + 1) = 0$$

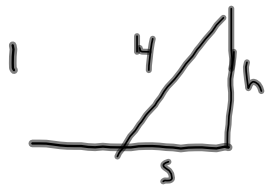
$$\cos 2\theta = \frac{-1 \pm \sqrt{1+8}}{-4} = \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$$

$$\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = \frac{2\pi}{3}, \quad \theta = \frac{\pi}{3}$$

$$A = r^2 \left(\sin \frac{2\pi}{3} - \sin \frac{2\pi}{3} \cos \frac{2\pi}{3}\right)$$

$$= r^2 \left(\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\right) = \underline{r^2 \frac{3}{4}\sqrt{3}}$$

7.2 13, 5, 7, 9, 13



$\frac{ds}{dt} = 0.5$

$h^2 + s^2 = 4^2$

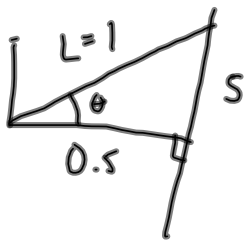
$h^2 = 4^2 - s^2 \qquad h = \sqrt{16 - s^2}$

$\frac{dh}{dt} = \frac{1 \cdot 2s}{2\sqrt{16 - s^2}} \frac{ds}{dt} = \frac{s}{\sqrt{16 - s^2}} \cdot \frac{ds}{dt}$

$s = 2 \quad \frac{ds}{dt} = 0.5 = \frac{2 \cdot 0.5}{\sqrt{16 - 4}} = \frac{1}{\sqrt{12}}$

$= \underline{\frac{1}{2\sqrt{3}}}$

3



$s = 0.5 \tan\theta$

$\frac{0.5}{L} = \cos\theta = \frac{0.5}{1} = \frac{1}{2}$

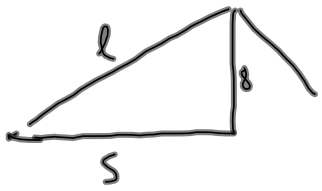
nèi $\theta = \frac{\pi}{3}$.

$\frac{d\theta}{dt} = 4\pi$

$\frac{ds}{dt} = 0.5 \cdot \frac{1}{\cos^2\theta} \cdot \frac{d\theta}{dt}$

$= 0.5 \cdot 4 \cdot 4\pi = \underline{8\pi}$ kmms^{-1}

5



$\frac{ds}{dt} = 4\ ms^{-1}$

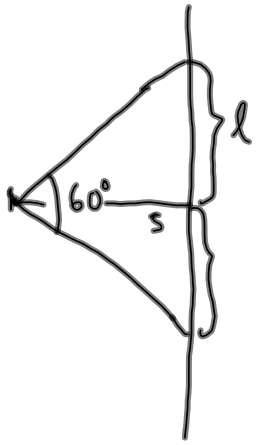
$l^2 = 8^2 + s^2$

$l = \sqrt{s^2 + 64}$ $\quad l = 10 \quad s = 6$

$\frac{dl}{dt} = \frac{s}{\sqrt{s^2+64}} \cdot \frac{ds}{dt}$

$\left.\frac{dl}{dt}\right|_{s=6} = \frac{6}{10} \cdot 4\ ms^{-1} = \underline{2.4\ ms^{-1}}$

7.



$$\frac{ds}{dt} = -1 \text{ m s}^{-1}$$

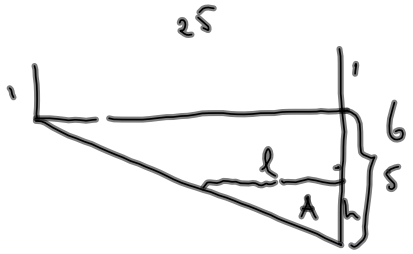
$$\frac{l}{s} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$l = \frac{1}{\sqrt{3}} \cdot s$$

$$\frac{dl}{dt} = \frac{1}{\sqrt{3}} \cdot \frac{ds}{dt} = -\frac{1}{\sqrt{3}}$$

$$\frac{d\,2l}{dt} = 2\frac{dl}{dt} = -\frac{2}{\sqrt{3}} \text{ m s}^{-1}$$

9



$\frac{dV}{dt} = 2\ m^3\ min^{-1}$

$V = 10A = 25h^2$

$\frac{h}{l} = \frac{5}{25}$

$A = \frac{lh}{2} = \frac{5}{2}h^2$

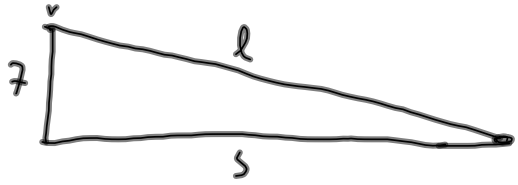
$l = 5h$

$\frac{dA}{dt} = 5h \cdot \frac{dh}{dt}$, $\frac{dV}{dt} = 50h\frac{dh}{dt}$

$h = 3$ $\quad 2 = 50 \cdot 3 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{75}\ m\ min^{-1}$

13



$l^2 = s^2 + 49$

$l = \sqrt{s^2+49} \qquad s = \sqrt{l^2-49}$

$\frac{dl}{dt} = 30\ ms^{-1} \qquad s = 24 \Rightarrow l = \sqrt{24^2+49}$

$= \sqrt{625} = 25$

$$\frac{ds}{dt} = \frac{l}{\sqrt{l^2-49}} \cdot \frac{dl}{dt} = \frac{25}{24} \cdot 30 = \frac{25 \cdot 5}{4} = \frac{125}{4}$$

7.4

5. $f(x) = \tan 2x \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$

$f'(x) = \frac{2}{\cos^2 2x} > 0$

$g = f^{-1} \qquad g(1) = y \iff f(y) = 1$

$y = \frac{\pi}{8}$

$g'(1) = \frac{1}{f'\left(\frac{\pi}{8}\right)} = \frac{1}{\frac{2}{\frac{1}{2}}} = \frac{1}{4}$

8 $y = g(x)$ är invers till f

$\Rightarrow \quad f(g(x)) = x$

$\frac{d}{dx}$: $\quad f'(g(x)) \cdot g'(x) = 1 \qquad g'(x) = \frac{1}{f'(g(x))}$

$$f''(g(x)) \cdot g'(x) \cdot g'(x) + f'(g(x)) \cdot g''(x) = 0$$

$$g''(x) = -\frac{f''(g(x)) \, (g'(x))^2}{f'(g(x))}$$

$f(x) = \sin x$

$g''\left(\frac{1}{2}\right)$

$g\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$$= -\frac{f''(g(x)) \, g'(x)}{\left(f'(g(x))\right)^2} = -\frac{f''(g(x))}{f'(g(x))^3}$$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$$= -\frac{-\sin\frac{\pi}{6}}{\left(\cos\frac{\pi}{6}\right)^3} = \frac{\frac{1}{2}}{\left(\frac{1}{2}\sqrt{3}\right)^3} = \frac{4}{3\sqrt{3}}$$

7.5 36

$$\lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\frac{\pi}{2} - x} =$$

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\frac{\cot x}{\frac{\pi}{2} - x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\frac{\pi}{2} - x} = \frac{1}{\sin x} \cdot \frac{\sin\left(\frac{\pi}{2} - x\right)}{\frac{\pi}{2} - x}$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\frac{\pi}{2} - x} = \lim_{x \to \frac{\pi}{2}} \frac{1}{\sin x} \cdot \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\frac{\pi}{2} - x}$$

$$= 1 \cdot 1 = \underline{\underline{1}}$$

7.1 36

$$\lim_{x \to 0} \frac{\arcsin x}{\sin 3x}$$

$$= \lim_{x \to 0} \frac{\arcsin x}{\sin 3x} \overset{\text{l'H}}{=} \lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{3\cos 3x} = \frac{1}{3}$$