4.3   1, 3abd, 4, 11, 13, 14, 15

1a $\lim_{n\to\infty} \frac{8n^4+2n}{3n^4-7}$

$$\frac{8n^4+2n}{3n^4-7} = \frac{8+2\frac{n}{n^4}}{3-7\frac{1}{n^4}} = \frac{8+\frac{2}{n^3}}{3-\frac{7}{n^4}}$$

$$\lim_{n\to\infty} \frac{8+\frac{2}{n^3}}{3-\frac{7}{n^4}} = \frac{8}{3} \quad \text{så} \quad \lim_{n\to\infty} \frac{8n^4+2n}{3n^4-7} = \underline{\frac{8}{3}}$$

sep 22-12:10

1e $\lim_{n\to\infty} \frac{n^5 + 2\sin n}{e^{-n} + 6n^5}$

$$\lim_{n\to\infty} \frac{n^5 + 2\sin n}{e^{-n} + 6n^5} = \lim_{n\to\infty} \frac{1 + \frac{2}{n^5}\sin n}{\frac{1}{n^5}\cdot e^{-n} + 6} = \underline{\underline{\frac{1}{6}}}$$

<u>3a</u> $\lim_{n\to\infty} \left(\sqrt{n+2} - \sqrt{n}\right) =$

$$\lim_{n\to\infty} \frac{(\sqrt{n+2} - \sqrt{n})(\sqrt{n+2} + \sqrt{n})}{(\sqrt{n+2} + \sqrt{n})} = \lim_{n\to\infty} \frac{n+2-n}{\sqrt{n+2} + \sqrt{n}} = \lim_{n\to\infty} \frac{2}{\sqrt{n+2} + \sqrt{n}} = 0$$

3d $\lim_{n\to\infty}\left(\sqrt{1+e^{-2n}} - e^{-n}\right)$

$\lim_{n\to\infty}\sqrt{1+e^{-2n}} = 1$ og $\lim_{n\to\infty} e^{-n} = 0$

så $\lim_{n\to\infty}\left(\sqrt{1+e^{-2n}} - e^{-n}\right) = 1$

sep 22-12:30

4 $\lim_{n\to\infty} a_n = L$ dersom det for hver $\varepsilon > 0$

finnes en N slik at $|a_n - L| < \varepsilon$ når $n > N$

b) $\lim_{n\to\infty} \frac{2\sin n}{n} = 0$

Gitt $\varepsilon > 0$

$$\left|\frac{2\sin n}{n} - 0\right| = \frac{2\,|\sin n|}{n} \leq \frac{2}{n} < \varepsilon$$

når $n > \frac{2}{\varepsilon}$

Så hvis $N > \frac{2}{\varepsilon}$ og $n > N$ så er $\left|\frac{2\sin n}{n}\right| < \varepsilon$.

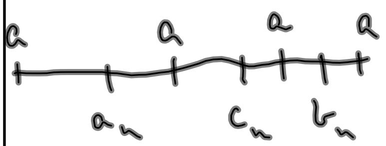
sep 22-12:34

11 $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = a$

$a_n \le c_n \le b_n$ for alle n

$$|c_n - a| \le \max\{|a_n - a|, |b_n - a|\}$$

Gitt $\varepsilon > 0$ La N_a og N_b



være slik at $|a_n - a| < \varepsilon$ og $|b_n - a| < \varepsilon$

når $n > N_a$ og $n > N_b$

Velg $N = \max\{N_a, N_b\}$.

13 $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$

$\underline{b}$ $a_n = \frac{1}{n}$ $b_n = \frac{1}{n^2}$

si tr $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} n = \infty$

14 $\lim_{n\to\infty} a_n = \infty = \lim_{n\to\infty} b_n$

$\underline{b}$

$a_n = n$ $b_n = n^2$

si tr $\lim_{n\to\infty} a_n - b_n = \lim_{n\to\infty} n - n^2 = -\infty$

sep 22-12:48

15 $a_n \geq a_{n+1}$ for alle n

$\{a_n, n=1,2,\dots\}$ er begrenset

det vil si at det fins en L slik at

$a_n > L$ for alle n.

La $a = \inf\{a_n\}$

La $\varepsilon > 0$, da er $a+\varepsilon$ ikke en nedre grense,

så det fins en N slik at $a_N < a+\varepsilon$.

Hvis nå $n > N$ så er $a+\varepsilon > a_N \geq a_n \geq a$

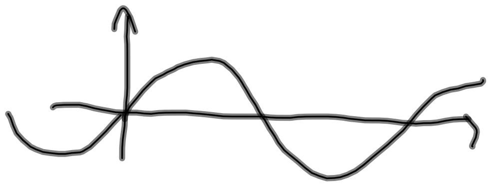
altså $|a_n - a| < \varepsilon$.

5.1 1abc 3ab 5ab e g 6ab 7 9abc

1 b $f(x) = \ln(x^2 - 4)$ $x^2 - 4 > 0$

så $D_f = (-\infty, -2) \cup (2, \infty)$

3 b $f(x) = \sin x^2$



$V_f = [-1, 1]$

sep 22-13:01

5 $f(x)$ er kontinuerlig i $x=a$ dersom $a \in D_f$ og det for hver $\varepsilon > 0$ finis en $\delta > 0$ slik at

$$|f(x) - f(a)| < \varepsilon \quad \text{når} \quad |x-a| < \delta$$

e) $f(x) = \frac{1}{x}$ $\quad x = 1$

Gitt $\varepsilon > 0$, skal finne passe δ.

$$\left|\frac{1}{x} - 1\right| = \left|\frac{1-x}{x}\right| = \frac{|x-1|}{|x|} < 2|x-1|$$

$|x| > \frac{1}{2}$

Velg $\delta < \min\left\{\frac{1}{2}, \frac{1}{2}\varepsilon\right\}$.

Hvis $|x-1| < \delta$ så er $\left|\frac{1}{x} - 1\right| < 2|x-1| < 2\delta < \varepsilon$

sep 22-13:22

g) $f(x) = \sqrt{x}$ $\quad x = 4$ $\quad x > 0$

$\varepsilon > 0$ $\quad |\sqrt{x} - \sqrt{4}| = \left|\frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{\sqrt{x} + 2}\right|$

$= \frac{|x-4|}{|\sqrt{x}+2|} < \frac{1}{2}|x-4|$

Velg $\delta = 2\varepsilon$ $\quad$ så vil $\overset{x>0}{}$ $|\sqrt{x} - \sqrt{4}| < \frac{1}{2}|x-4| < \frac{1}{2}\cdot 2\varepsilon = \varepsilon$

når $|x-4| < \delta$

sep 22-13:30

6 b

$$f(x) = \begin{cases} \cos \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\cos \frac{1}{\frac{1}{2k\pi}} = \cos 2k\pi = \underline{1} \qquad k = 1, 2, 3 \ldots$$

La $\varepsilon = \frac{1}{2}$, da fins det for hver

$\delta > 0$ en k slik at $0 < \frac{1}{2k\pi} < \delta$, så

$$f\left(\frac{1}{2k\pi}\right) = 1 > \frac{1}{2} \quad \text{selv om}$$

$$0 < \frac{1}{2k\pi} < \delta .$$

Så f er diskontinuerlig i $x = 0$.

sep 22-13:35

d $\quad f(x) = \cos \ln |\sin(e^{x^2})| \qquad x = 0$

x^2 e kontinuerlig i $x = 0$

e^u er kontinuerlig i $u = 0$

så e^{x^2} — , — $x = 0 \qquad e^0 = 1$

$\sin u$ — , — i $u = 1$

så $\sin(e^{x^2})$ — , — i $x = 0 \qquad \sin 1 > 0$

$\ln|u|$ — , — i $u = \sin 1$

så $\ln|\sin(e^{x^2})|$ — , — i $x = 0$

$\cos u$ — , — i $\ln|\sin 1|$

så $f(x)$ — , — i $x = 0$

sep 22-13:41

9a $f(x) = x^3$ ingen diskontinuiteter

b $f(x) = \begin{cases} \sqrt{x} & x > 0 \\ x+1 & x \leq 0 \end{cases}$

eneste mulige diskontinuitet er i $x = 0$

$f(0) = 1$

Velg $\varepsilon = \frac{1}{2}$ da er $\sqrt{x} < \frac{1}{2}$ når $0 < x < \frac{1}{4}$

så $|\sqrt{x} - 1| > \frac{1}{2}$

så f er disk. i $x = 0$

sep 22-13:46

9c $\quad f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$\sin \frac{1}{\frac{1}{\frac{\pi}{2} + 2k\pi}} = \sin\left(\frac{\pi}{2} + 2k\pi\right) = 1$$

så med $\varepsilon = \frac{1}{2}$ så fins det for hver

$\delta > 0$ en k slik at $0 < \frac{1}{\frac{\pi}{2} + 2k\pi} < \delta$

mens $f\left(\frac{1}{\frac{\pi}{2} + 2k\pi}\right) = 1 > \frac{1}{2} = \varepsilon$

så f er disk. i $x = 0$.