1.3 4(c) (fra sist uke)

$$\vec{x}, \vec{y} \in \mathbb{C}^n \qquad \vec{x} = (\underbrace{a_1 + ib_1}_{x_1}, \ldots, \underbrace{a_i + ib_i}_{x_i}, \ldots, \underbrace{a_n + ib_n}_{x_n})$$

$$\vec{y} = (c_1 + id_1 \; - - - \; c_n + id_n)$$

$$\vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i \overline{y_i} \qquad |\vec{x}|^2 = \sum_{i=1}^{n} |x_i|^2$$

$$|x - y|^2 = |(x_1 - y_1, \; , x_n - y_n)|^2 = \sum |x_i - y_i|^2 = \sum |(a_i + ib_i - c_i - id_i)|^2$$

$$= \sum_i \left((a_i - c_i)^2 + (b_i - d_i)^2\right) = \sum_i \left(a_i^2 + c_i^2 + b_i^2 + d_i^2\right)$$

$$- \sum_i (2a_i c_i + 2b_i d_i)$$

$$= \sum_i a_i^2 + c_i^2 + \sum_i b_i^2 + d_i^2 - 2 \sum (a_i c_i + b_i d_i)$$

$$= \sum |x_i|^2 + \sum |y_i|^2 - 2 \sum Re(x_i \overline{y_i})$$

$$x_i \overline{y_i} = (a_i + ib_i) \cdot (c_i - id_i)$$
$$= a_i c_i + b_i d_i - i(a_i d_i - b_i c_i)$$

$$= |\vec{x}| + |\vec{y}| - 2 R(\vec{x} \cdot \vec{y})$$

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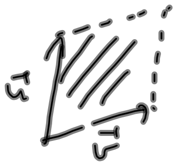
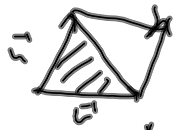
1.4   1, 2, 3, 4, 6, 7, 9, 10

2) 

$A = |\vec{u} \times \vec{v}|$



$A_T = \frac{1}{2}|\vec{u} \times \vec{v}|$

$\vec{a} = (-2, 3, 1)$, $\vec{b} = (4, 0, -2)$

$$A = \Bigg|\underbrace{\begin{vmatrix} i & j & k \\ -2 & 3 & 1 \\ 4 & 0 & -2 \end{vmatrix}}_{\vec{a} \times \vec{b}}\Bigg| = |(-6, 0, -12)| = \sqrt{36 + 144} = \sqrt{180}$$

$$= 3 \cdot 2 \cdot \sqrt{5} = \underline{6\sqrt{5}}$$

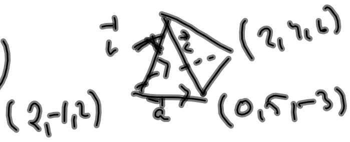
4) $\vec{v} \perp (2, 0, -3)$

$\vec{v} \perp (-1, 3, 4)$

$(3, -2, 4)$

$\vec{v}$ parallell med $\begin{vmatrix} i & j & k \\ 2 & 0 & -3 \\ -1 & 3 & 4 \end{vmatrix} = (9, -5, 6)$

kan velge $\underline{\vec{v} = (9, -5, 6)}$

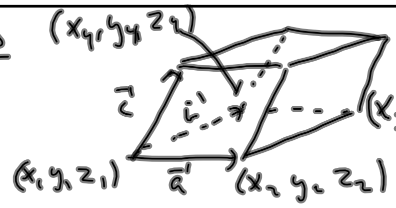
7) 

$V = \frac{1}{2}\frac{1}{3}|(\vec{a} \times \vec{b}) \cdot \vec{c}|$

$\vec{a} = (-2, 6, -5)$, $\vec{b} = (1, -1, 2)$, $\vec{c} = (0, 5, 4)$

$$V = \frac{1}{6}\left|\begin{vmatrix} -2 & 6 & -5 \\ 1 & -1 & 2 \\ 0 & 5 & 4 \end{vmatrix}\right| = \frac{1}{6}\left|-2(-14) - 1 \cdot 49\right| = \frac{21}{6} = \underline{\frac{7}{2}}$$

1.4 10



$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

hvis alle x_i y_i z_i er hele tall, så er koordinatene til $\vec{a}$, $\vec{b}$ og $\vec{c}$ også hele tall, så alle tallene i (entriene) determinanten som gir volumet er også hele tall. Siden en determinant er en sum av produkt av entriene, så er også determinanten et helt tall.

1.5   1, 3, 5, 7, 8, 10, 11, 12

5a $A = \begin{pmatrix} 1 & 0 & -3 \\ -2 & -3 & 2 \end{pmatrix}$ $x = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$

$$A \cdot x = \begin{pmatrix} 1 & 0 & -3 \\ -2 & -3 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1\cdot(-2) + 0\cdot 3 + (-3)\cdot(-1) \\ -2(-2) + (-3)\cdot 3 + 2\cdot(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -7 \end{pmatrix}$$

7

	x	y	z
X	0.7	0.1	0.2
Y	0.3	0.5	0.2
Z	0.4	0.2	0.4

$$\begin{pmatrix} 0.7 & 0.3 & 0.4 \\ 0.1 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.4 \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} 0.7X_0 + 0.3Y_0 + 0.4Z_0 \\ 0.1X_0 + 0.5\cdot Y_0 + 0.2Z_0 \\ 0.2X_0 + 0.2Y_0 + 0.4Z_0 \end{pmatrix} = \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}$$

$X_0 = 50$
$Y_0 = 70$
$Z_0 = 80$

$$\Rightarrow \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \begin{pmatrix} 0.7\cdot 50 + 0.3\cdot 70 + 0.4\cdot 80 \\ 0.1\cdot 50 + 0.5\cdot 70 + 0.2\cdot 80 \\ 0.2\cdot 50 + 0.2\cdot 70 + 0.4\cdot 80 \end{pmatrix}$$

$$= \begin{pmatrix} 88 \\ 56 \\ 56 \end{pmatrix}$$

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1.5 10

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0.1 & 0 & 0.2 \\ 0.2 & 0.3 & 0.2 \\ 0.3 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.4 \end{pmatrix} \end{array} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{array}{c} 10 \\ 12 \\ 8 \end{array} = \begin{pmatrix} 0.1x + 0 \cdot y + 0.2z \\ \\ \\ \end{pmatrix}$$

$x = 10, \; y = 12, \; z = 8$

$$\rightarrow \begin{pmatrix} 2.6 \\ 7.2 \\ 7.4 \\ 7.4 \\ 5.4 \end{pmatrix}$$

11

$$\begin{array}{ll|c} & & \begin{array}{ccc} sm & s & im \end{array} \\ \text{smittebærere (sm)} & & \\ \text{syke} & s & \\ \text{immune} & im & \end{array}$$

$$\begin{pmatrix} 0.94 & 0 & 0.01 \\ 0.05 & 0.2 & 0 \\ 0.01 & 0.8 & 0.99 \end{pmatrix} \begin{pmatrix} sm_0 \\ s_0 \\ im_0 \end{pmatrix}$$

$$\overset{\shortparallel}{A}$$

a)

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}$$

$x_0 = 90$

$y_0 = 10$

$z_0 = 0$

$$\begin{pmatrix} 0.94 & 0 & 0.01 \\ 0.05 & 0.2 & 0 \\ 0.01 & 0.8 & 0.99 \end{pmatrix} \begin{pmatrix} 90 \\ 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 84.6 \\ 6.5 \\ 8.9 \end{pmatrix}$$

$$\begin{pmatrix} \quad \end{pmatrix} \begin{pmatrix} 84.6 \\ 6.5 \\ 8.9 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

1.6 1, 4, 6, 10, 11, 12

4\. a) AB:

A 8×6

B 6×9

AB: $(8 \times 6) \cdot (6 \times 9) = (8 \times 9)$ - matrise

b) A: 4×3

AB 4×5

$\Rightarrow$ B? AB: $(4 \times 3)(3 \times 5)$

så B er en (3×5) - matrise.

c) $A \cdot B$ 5×7 matrise, da er

A en $(5 \times s)$ - matrise og

B en $(s \times 7)$ - matrise

$$\underbrace{((5 \times s)(s \times 7))}_{AB}$$

, så B har 7 søyler.

6. $A = \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$ $D = \begin{pmatrix} 3 & 7 \\ 0 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$$

$$AC = \begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$$

$$AD = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\underline{10}$ $A^k = \underbrace{A \cdot A \cdot A \cdot \quad \cdot A}_{k}$

$A^{(n\times n)}$

$$A = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} \qquad A^2 = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -11 \\ -5 & 7 \end{pmatrix}$$

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12

c)

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0.7 & 0.05 & 0.10 \\ B & 0.15 & 0.75 & 0.15 \\ C & 0.1 & 0.1 & 0.7 \end{array} \qquad \overset{u}{\underset{\parallel}{}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

M

$$Ku = N \cdot \underbrace{Mu}_{v}$$

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 1 & 0.2 & 0 \\ B & 0 & 0.8 & 0.05 \\ C & 0 & 0 & 0.95 \end{array} \qquad \overset{v}{} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

N

c) $K^{3\times3} = NM$

$K\begin{pmatrix} 200 \\ 300 \\ 400 \end{pmatrix}$ = fordeling på slutten av dagen

d) $K^2(\quad)$ etter neste dag.

1.7 1 3 4 8 10

4 $A^{-1} = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \quad \left(= \frac{1}{\det A} \cdot \begin{pmatrix} - & - \\ & -- \end{pmatrix} \right)$

$A = (A^{-1})^{-1} = \frac{1}{1} \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$

10 $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} ax+bz & ay+bw \\ cx+dz & cy+dw \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$d \cdot \quad ax+bz = 1 \qquad ay+bw = 0$

$-b \cdot \quad cx+dz = 0 \qquad cy+dw = 1$

$adx + bdz = d$

$-bcx + (-bdz) = 0$

$(ad-bc)x = d$ løsning dersom $(ad-bc) \neq 0$

etc $\longrightarrow$.

$\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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8. A B har invers.

$$(AB)^T = B^T A^T$$

$$A A^{-1} = id = I$$

$$(A A^{-1})^T = id = I$$

$$\shortparallel$$

$$(A^{-1})^T \cdot A^T$$

$$(A^{-1})^T$$

$$\shortparallel$$

$$B^T \underbrace{A^T \cdot (A^T)^{-1}} (B^T)^{-1}$$

$$= B^T I (B^T)^{-1}$$

$$= B^T \cdot (B^T)^{-1} = \underline{I}$$

$$\Rightarrow ((AB)^T)^{-1} = \underline{(A^T)^{-1} \cdot (B^T)^{-1}}$$

$$= \underline{(A^{-1})^T \cdot (B^{-1})^T}$$

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