Plenum 16/11-12

1.1: 1, 2, 3 (a), 4 (d), 5

1.2: 1, 3, 5, 7, 11, 13, 15, 17, 19, 21, 25, 27

1.3: 1, 3, 4

1.4: 1a, 2, 3, 4, 6, 7, 9, 10

1.5: 1, 3, 5, 7, 8, 10, 11, 12

litt

FVLA

1.1: 3) a) Vis: For alle $\vec{x}, \vec{y} \in \mathbb{R}^n$ er

$$(\vec{x}+\vec{y})\cdot(\vec{x}+\vec{y}) = \vec{x}\cdot\vec{x} + 2\vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{y}:$$

$$\underbrace{(\vec{x}+\vec{y})}_{:=\vec{z}}\cdot(\vec{x}+\vec{y}) \underset{\substack{\vec{z}:=\vec{x}+\vec{y},\\ \text{bruk Set. 1.1.1e}}}{=} \vec{z}\cdot\vec{x} + \vec{z}\cdot\vec{y} = (\vec{x}+\vec{y})\cdot\vec{x} + (\vec{x}+\vec{y})\cdot\vec{y}$$

$$\underset{\text{Set. 1.1.1e}}{=} \vec{x}\cdot\vec{x} + \vec{y}\cdot\vec{x} + \vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{y}$$

$$\underset{\text{Set. 1.1.1b)}}{=} \vec{x}\cdot\vec{x} + \vec{x}\cdot\vec{y} + \vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{y}$$

$$= \vec{x}\cdot\vec{x} + 2\vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{y}$$

4.) d): Vis at $(s+t)\vec{a} = s\vec{a} + t\vec{a}$, $\vec{a} \in \mathbb{R}^n$, $s, t \in \mathbb{R}$

$$(s+t)\vec{a} = (s+t)(a_1, a_2, \dots, a_n)$$

$$= ((s+t)a_1, (s+t)a_2, \dots, (s+t)a_n)$$

$$= (sa_1 + ta_1, sa_2 + ta_2, \dots, sa_n + ta_n)$$

$$= (sa_1, sa_2, \dots, sa_n) + (ta_1, ta_2, \dots, ta_n)$$

$$= s(a_1, a_2, \dots, a_n) + t(a_1, a_2, \dots, a_n)$$

$$= s\vec{a} + t\vec{a} \quad \boxtimes$$

5.) Grossistfirma; n vareslag:

m_1 enheter av vare 1, verdi for enhet p_1

m_2 —— " —— 2, — " —— p_2

⋮ ⋮

m_n —— " —— n, — " —— p_n

Total verdi av varelager:

$$\begin{aligned} V_{lager} &= m_1 p_1 + m_2 p_2 + \dots + m_n p_n \\ &= (m_1, m_2, \dots, m_n) \cdot (p_1, p_2, \dots, p_n) \\ &:= \vec{m} \cdot \vec{p} \end{aligned}$$

1.2: 7) $\vec{a} = (4,3)$; skriv $\vec{a}$ som sum av $\vec{b}$ og $\vec{c}$ der $\vec{b}$ er parallell m/ $\vec{d} = (1,2)$, og $\vec{c} \perp \vec{d}$.

Vis: $(4,3) = \vec{b} + \vec{c} = t(1,2) + \vec{c}$

siden $\vec{b}$ er parallell m/ $\vec{d}$

$$0 = \vec{c} \cdot \vec{d} = (c_1, c_2) \cdot (1,2) = c_1 + 2c_2$$

$$\Downarrow$$

$$c_1 = -2c_2$$

Vil finne: $(4,3) = (t, 2t) + (-2c_2, c_2)$

$$\Downarrow$$

$$4 = t - 2c_2 \Rightarrow t = 4 + 2c_2$$

$$3 = 2t + c_2$$

$$\Downarrow$$

$$3 = 8 + 4c_2 + c_2$$

$$\Downarrow$$

$$5c_2 = -5$$

$$\underline{c_2 = -1}$$

$$\Rightarrow t = 4 + 2(-1) = \underline{2}$$

Så: $\vec{a} = \underbrace{(2,4)}_{\vec{b}} + \underbrace{(2,-1)}_{\vec{c}}$

15.) Vis: For alle $\vec{x}, \vec{y} \in \mathbb{R}^n$ er $|\vec{x}| - |\vec{y}| \leq |\vec{x} - \vec{y}|$:

$$|\vec{x}| = |(\vec{x} - \vec{y}) + \vec{y}| \underset{\boxed{\Delta\text{-ulikhet}}}{\leq} |\vec{x} - \vec{y}| + |\vec{y}|$$

$$\Downarrow$$

$$|\vec{x}| - |\vec{y}| \leq |\vec{x} - \vec{y}| \quad \boxtimes$$

Vis: $|\vec{y}| - |\vec{x}| \leq |\vec{x} - \vec{y}|$:

Bytt rollene til $\vec{x}$ og $\vec{y}$ over og bruk at $|\vec{y} - \vec{x}| = |\vec{x} - \vec{y}|$. $\boxtimes$

Dermed er $\big||\vec{x}| - |\vec{y}|\big| \leq |\vec{x} - \vec{y}|$

fordi: $|\vec{x}| - |\vec{y}| \leq |\vec{x} - \vec{y}|$ og $-(|\vec{x}| - |\vec{y}|)$
$= |\vec{y}| - |\vec{x}| \leq |\vec{x} - \vec{y}|$ (fra def. av abs. verdi). $\boxtimes$

19.) Linje gj. $\overset{\vec{a}=}{(-3,-2,5,8)}$, parallell m/ $\overset{\vec{b}=}{(1,-2,-1,3)}$:

$$\vec{r}(t) = \vec{a} + t\vec{b} = (-3,-2,5,8) + t(1,-2,-1,3)$$

$$= (t-3, -2-2t, 5-t, 8+3t)$$

$(1,-6,3,14)$ på linjen? Fins t s.a.

$1 = t-3$, $-6 = -2-2t$, $3 = 5-t$, $14 = 8+3t$?

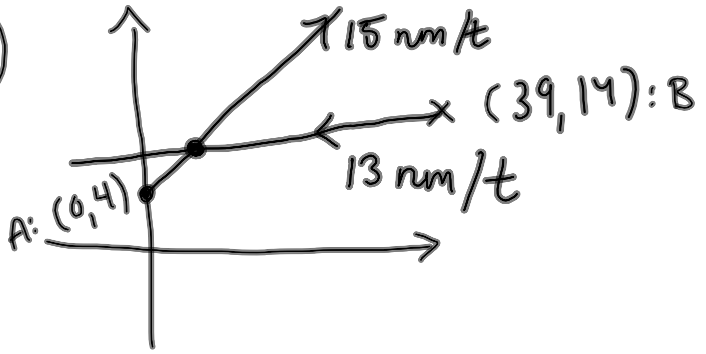
$\Downarrow$ $t = 4$

$\Downarrow$ Holder ikke for $t=4$

$\Downarrow$

Nei! $\underline{\underline{(1,-6,3,14)}}$ er ikke på linja.

2.5.)



a) Parameterframstillinger:

A: $\vec{r}_A(t) = (0,4) + t(3,4) = (3t, 4+4t)$

B: $\vec{r}_B(t) = (39,14) + t(-12,5) = (39-12t, 14+5t)$

Kryss: $3t_1 = 39 - 12t_2$, $4+4t_1 = 14+5t_2$

$\Downarrow$ (Løs!)

$t_1 = 5$, $t_2 = 2$

De krysser hverandre i $(3\cdot 5, 4+4\cdot 5) = (15, 24)$.

b) Kolliderer? Nei! Båtene krysser kun én gang.

A må flytte seg: $\sqrt{(15-0)^2 + (24-4)^2} = 25$ nm.

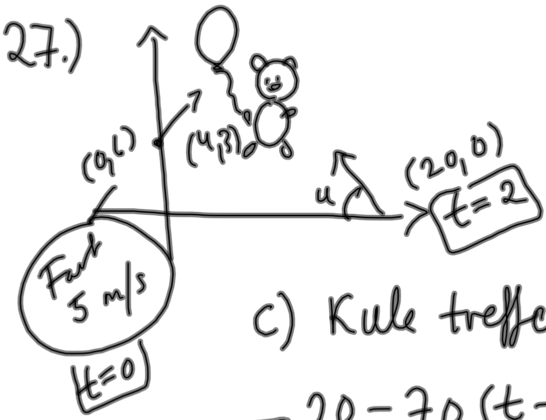
B må flytte seg: $\sqrt{(39-15)^2 + (14-24)^2} = 26$ nm.

A bruker: $\frac{25}{15} = \frac{5}{3}$ timer

B —"— : $\frac{26}{13} = 2$ timer

Så, siden tid A bruker $\neq$ tid B bruker, vil de ikke kræsje.

27.)



c) Kule treffer dersom det fins t s.a.

$$20-70(t-2)\cos u = 4t$$

$$70(t-2)\sin u = 6+3t$$

$$\to \quad t = \frac{140\sin(u)+6}{70\sin(u)-3} \qquad (\star)$$

$$20-\left(70\,\frac{140\sin u+6}{70\sin u-3}-140\right)\cos u = 4\,\frac{140\sin u+6}{70\sin u-3}$$

$$1400\sin u-60-(70\cdot 6+140\cdot 3)\cos u = 4(140\sin u+6)$$

$$60(\sin u-\cos u)=6$$

$$\underbrace{\frac{1}{\sqrt{2}}}_{\cos\frac{\pi}{4}}\sin-\underbrace{\frac{1}{\sqrt{2}}}_{\sin\frac{\pi}{4}}\cos u=\frac{1}{10\sqrt{2}}$$

$$\Downarrow \quad \left(\cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}\right)$$

$$\sin\left(u-\frac{\pi}{4}\right)=\frac{1}{10\sqrt{2}}$$

$$u-\frac{\pi}{4}=\arcsin\left(\frac{1}{10\sqrt{2}}\right)$$

$$u=\frac{\pi}{4}+\arcsin\left(\frac{1}{10\sqrt{2}}\right)$$

$$\approx \underline{\underline{49^\circ}}$$

Så K-R må skyte m/ vinkel $u\approx 49^\circ$.

Hvor langt til bakken?

Når treffes ballongen?

$$t^* = \frac{140 \sin 49° + 6}{70 \sin 49° - 3} \approx 2,24$$

(*)

$$\Downarrow$$

Høyde over bakken $= 6 + 3 \cdot t^* = \underline{12,7 \text{ m}}$

a): y-koordinat til ballongen

1.3: 4.) Vis: For alle $\vec{x}, \vec{y} \in \mathbb{C}^n$,

$|\vec{x}-\vec{y}|^2 = |\vec{x}|^2 - 2\,\mathrm{Re}(\vec{x}\cdot\vec{y}) + |\vec{y}|^2$:

$$|\vec{x}-\vec{y}|^2 = (\vec{x}-\vec{y})\cdot(\vec{x}-\vec{y}) = \vec{x}\cdot\vec{x} - \vec{x}\cdot\vec{y} - \vec{y}\cdot\vec{x} + \vec{y}\cdot\vec{y}$$

$$= |\vec{x}|^2 + |\vec{y}|^2 - \vec{x}\cdot\vec{y} - \overline{\vec{x}\cdot\vec{y}}$$

$$= |\vec{x}|^2 + |\vec{y}|^2 - 2\,\mathrm{Re}(\vec{x}\cdot\vec{y})$$

NB. $\vec{x}\cdot\vec{y} \neq \vec{y}\cdot\vec{x}$ for $\vec{x}, \vec{y} \in \mathbb{C}^n$

1.4: 2) Areal for parallellogram utspent av

$\vec{a} = (-2, 3, 1)$ og $\vec{b} = (4, 0, -2)$?

$A = |a \times b|$

M: $a \times b$:

$$\begin{array}{cccccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & 1 & -2 & 3 & 1 \\ 4 & 0 & -2 & 4 & 0 & -2 \end{array}$$

$$-1\cdot 0\,\vec{i} - (-2)(-2)\vec{j} - 3\cdot 4\,\vec{k} + 3(-2)\vec{i} + 1\cdot 4\,\vec{j} + (-2)0\,\vec{k}$$

$$= (0, -4, 0) - (0, 0, 12) - (6, 0, 0) + (0, 4, 0)$$

$$= (-6, 0, -12)$$

$$A = |(-6, 0, -12)| = \sqrt{6^2 + 12^2} = \underline{\underline{6\sqrt{5}}}$$

9.) Ligning for planet gj. $\vec{a}=(1,2,1)$, $\vec{b}=(2,3,0)$, $\vec{c}=(2,1,-1)$:

$$\vec{b}-\vec{a}=(1,1,-1)\,,\quad \vec{c}-\vec{a}=(1,-1,-2)$$

$$\vec{n}=(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=(-3,1,-2)$$

normal-vektor for plan

$$\begin{array}{cccccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & k \\ 1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -2 & 1 & -1 & -2 \end{array}$$

Planligningen:

$(-3,1,-2)\cdot(x,y,z)=d$ for en eller annen $d\in\mathbb{R}$.

$$-3x+y-2z=d$$

<u>Bestemme d</u>: Setter inn $\vec{a}$-koordinatene

$$d=-3\cdot 1+2-2\cdot 1=-3$$

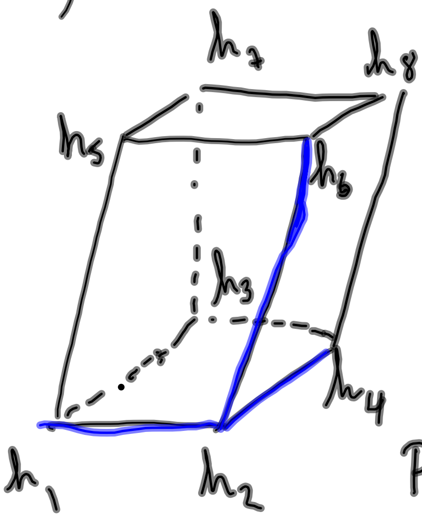
$$\Downarrow$$

Ligningen til planet:

$$-3x+y-2z=-3$$

$$(\,3x-y+2z=3\,)$$

10.) Et parallellepiped har 8 hjørner.



Kall disse $h_1, \dots, h_8$. Disse har heltallige koeff.

$$h_i = \Big(\underset{\in \mathbb{Z}}{k_1^{(i)}}, \underset{\in \mathbb{Z}}{k_2^{(i)}}, \underset{\in \mathbb{Z}}{k_3^{(i)}} \Big).$$

Parallellepipedet er utspent av (se fig.) $h_4 - h_2$, $h_1 - h_2$, $h_6 - h_2$.

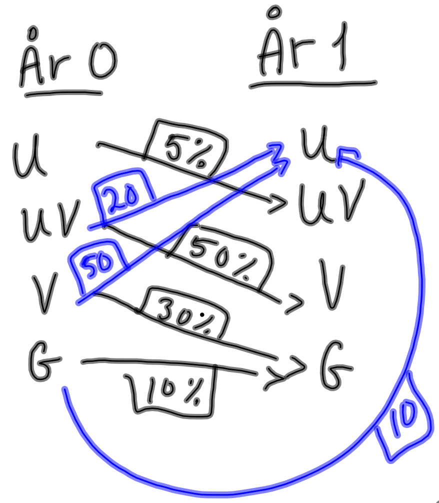
Volumet er da (fra Setning 1.4.4):

$$V = \underbrace{\Big| \underbrace{\Big(\underbrace{\underbrace{(h_4 - h_2)}_{\text{helt. koeff.}} \times \underbrace{(h_1 - h_2)}_{\text{helt. koeff.}}}_{\text{helt. koeff}} \Big) \cdot \underbrace{(h_6 - h_2)}_{\text{helt. koeff.}}}_{\text{helt.}} \Big|}_{\text{heltall}}$$

$\blacksquare$

1.5: 12.) Dyr: 4 grupper:

1) Unge (U)
2) Unge voksne (UV)
3) Voksne (V)
4) Gamle (G)



$$\vec{v}_n = \begin{bmatrix} x_n \\ y_n \\ z_n \\ u_n \end{bmatrix} = \begin{bmatrix} \#\text{ U n år} \\ \#\text{ UV} -"- \\ \#\text{ V} -"- \\ \#\text{ G} -"- \end{bmatrix}$$

a)

$$A = \begin{bmatrix} 0 & 20 & 50 & 10 \\ 0{,}05 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0 \\ 0 & 0 & 0{,}3 & 0{,}1 \end{bmatrix}$$

b) År 0: $\vec{v}_0 = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \end{bmatrix}$

År 1: $\vec{v}_1 = A\vec{v}_0 = \begin{bmatrix} 0 & 20 & 50 & 10 \\ 0{,}05 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0 & 0 \\ 0 & 0 & 0{,}3 & 0{,}1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5000 \\ 0 \\ 0 \\ 30 \end{bmatrix}}}$

År 2: $\vec{v}_2 = A\vec{v}_1 = \begin{bmatrix} 300 \\ 250 \\ 0 \\ 3 \end{bmatrix}$

Ved år 2 har vi 300 U, 250 UV, 0 V og 3 G.