$$= \frac{1}{13} \ln e^{x} - \frac{1}{26} \ln\left(e^{2x} + 4e^{x} + 13\right) - \frac{2}{39} \arctan\left(\frac{e^{x}+2}{3}\right) + C$$

$$= \frac{1}{13}\left(x - \frac{1}{2} \ln\left(e^{2x} + 4e^{x} + 13\right) - \frac{2}{3} \arctan\left(\frac{e^{x}+2}{3}\right)\right) + C$$

31.) $\int \ln(x^2 + 2x + 10)\,dx$

$$= x \ln(x^2 + 2x + 10) - \int \frac{x(2x+2)}{x^2 + 2x + 10}\,dx$$

Delvis int:
$u(x) = \ln(x^2 + 2x + 10)$
$v'(x) = 1$
$\Downarrow$
$u'(x) = \frac{2x+2}{x^2+2x+10}$
$v(x) = x$

$$= x \ln(x^2 + 2x + 10) - \int \frac{2x^2 + 2x}{x^2 + 2x + 10}\,dx$$

M: $\int \frac{2x^2 + 2x}{x^2 + 2x + 10}\,dx$

Polynomdiv: $2x^2 + 2x : x^2 + 2x + 10 = 2 - \frac{2x+20}{x^2+2x+10}$

$-(2x^2 + 4x + 20)$

$-2x - 20$

$$\int \frac{2x^2 + 2x}{x^2 + 2x + 10}\,dx = \int 2\,dx - \int \frac{2x + 20}{x^2 + 2x + 10}\,dx$$

$$= 2x - \int \frac{2x+2}{x^2+2x+10}\,dx - 18 \int \frac{1}{x^2 + 2x + 10}\,dx$$

$$= -\frac{18}{9} \int \frac{1}{\left(\frac{x+1}{3}\right)^2 + 1}\,dx - \int \frac{1}{u}\,du + 2x$$

$u = x^2 + 2x + 10$
$du = (2x+2)\,dx$

$$= -2 \arctan\left(\frac{x+1}{3}\right) \cdot 3 - \ln(x^2 + 2x + 10) + C$$

$$= -6\arctan\left(\frac{x+1}{3}\right) - \ln(x^2+2x+10) + C$$

Så:

$$\int \ln(x^2+2x+10)\,dx = x\ln(x^2+2x+10)$$
$$+ 6\arctan\left(\frac{x+1}{3}\right)$$
$$+ \ln(x^2+2x+10) - 2x + C$$

$$= (x+1)\ln(x^2+2x+10)$$
$$+ 6\arctan\left(\frac{x+1}{3}\right) - 2x + C$$

9.5: Uegentlige integraler

1) a) $\int_0^\infty \frac{1}{1+x^2}\,dx = \lim_{a\to\infty}\left[\arctan x\right]_{x=0}^{a}$

$$= \lim_{a\to\infty} \arctan a - \arctan 0$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

b) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \left[\arcsin x\right]_{x=0}^{1} = \arcsin 1 - \arcsin 0$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

siden ledende ledd er som $\frac{1}{x}$

3) a) $\int_0^\infty \frac{x+4}{x^2+2x+1}\,dx$ — Avgjør om integralet konvergerer eller divergerer

Sammenligner m/ $f(x) = \frac{1}{x}$ (som divergerer fra Set. 8.5

$$\lim_{x\to\infty} \frac{\frac{x+4}{x^2+2x+1}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{x^2+4x}{x^2+2x+1}$$

Deler på høyest potens av x

$$= \lim_{x\to\infty} \frac{1+\frac{4}{x}}{1+\frac{2}{x}+\frac{1}{x^2}} = 1 > 0 \Rightarrow \int_0^\infty \frac{x+4}{x^2+2x+1}\,dx \text{ divergerer}$$

fra grensesammenligningskriteriet