Plenumsregning 30/8-13

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Komplekse tall

3.1: 1 (d, g), 3 (c), 5 a c, 6, 8 (i), 9

3.2: 1 (a), 3 (c), 5 (b), 7a, 9, 10 (b, d), 13, 15

3.3: 1 (a), 3 (a), 7, 8

3.1: 1) d) $(5+2i)(3+i) = 5\cdot 3 + 5\cdot i + 2i\cdot 3 + 2i\cdot i$

$= 15 + 5i + 6i - 2 = \underline{\underline{13 + 11i}}$

g) $\dfrac{-5+2i}{5-4i} = \dfrac{(-5+2i)(5+4i)}{(5-4i)(5+4i)} = \dfrac{-25-20i+10i-8}{25+16}$

$= \dfrac{-33-10i}{41} = \underline{\underline{-\dfrac{33}{41} - \dfrac{10}{41}i}}$

3.) c) $\overline{-7-8i} = \underline{\underline{-7+8i}}$

5.) c) $\dfrac{z-2}{z+1} = 3i$

$$z-2 = 3i(z+1)$$

$$z-2 = 3iz + 3i$$

$$z - 3iz = 2 + 3i$$

$$z(1-3i) = 2+3i$$

$$z = \frac{2+3i}{1-3i} = \frac{(2+3i)(1+3i)}{(1-3i)(1+3i)} = \frac{2+6i+3i-9}{1+9}$$

$$= \frac{-7+9i}{10} = \underline{\underline{-\frac{7}{10} + \frac{9}{10}i}}$$

6.) $\begin{cases} z + w = 2i & (1) \\ z - w = 3+i & (2) \end{cases}$

(1)+(2): $z + \cancel{w} + z - \cancel{w} = 2i + 3 + i$

$$2z = 3i + 3$$

$$z = \underline{\underline{\frac{3}{2} + \frac{3}{2}i}}$$

Innsett i (1): $z + w = 2i$

$$\frac{3}{2} + \frac{3}{2}i + w = 2i$$

$$w = \underline{\underline{-\frac{3}{2} + \frac{1}{2}i}}$$

8.) i) Vis: $\overline{z} + \overline{w} = \overline{z+w}$:

La $z = a+ib$, der $a, b \in \mathbb{R}$. La
$w = c+id$, der $c, d \in \mathbb{R}$.

VS: $\overline{z} + \overline{w} = (a-ib) + (c-id) = \underline{(a+c) - i(b+d)}$

HS: $\overline{z+w} = \overline{(a+c)+i(b+d)} = \underline{(a+c) - i(b+d)}$

Ser at VS = HS, så regneregelen holder.

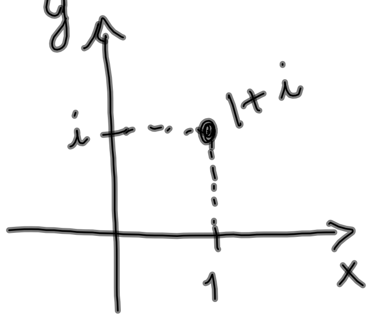
9.) $\overline{(\overline{z}\,w)} = \overline{(\overline{z})}\;\overline{w} = z\,\overline{w}$

3.1.5: Regneregler konjugasjon

$\overline{(\overline{z})} = z$ fordi:
La $z = a+ib$. Da er
$\overline{z} = a-ib$, og derfor er
$\overline{(\overline{z})} = \overline{a-ib} = a+ib = z$

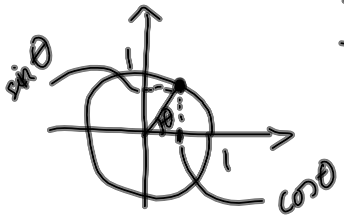
3.2: Geometrisk tolkning av komplekse tall

1) a) $1+i$:



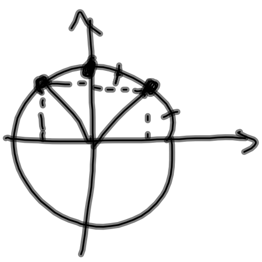
Tips for å huske eksakte verdier:

→ Tegn enhetssirkelen



$$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{2} = \frac{\cancel{\sqrt{2}}}{(\sqrt{2})^{\cancel{2}}} = \frac{1}{\sqrt{2}}$$



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

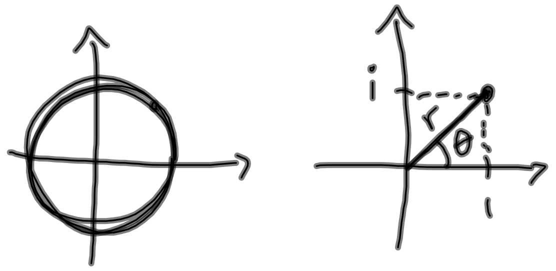
=

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

3.) c) $1+i$: $\quad r=\sqrt{1^2+1^2}=\underline{\sqrt{2}}$

$$\begin{aligned}\sqrt{2}\sin\theta &= 1\\ \sqrt{2}\cos\theta &= 1\end{aligned} \;\Rightarrow\; \begin{aligned}\sin\theta &= \frac{1}{\sqrt{2}}\\ \cos\theta &= \frac{1}{\sqrt{2}}\end{aligned} \;\Rightarrow\; \underline{\theta=\frac{\pi}{4}+2k\pi}$$

der $k\in\mathbb{Z}$

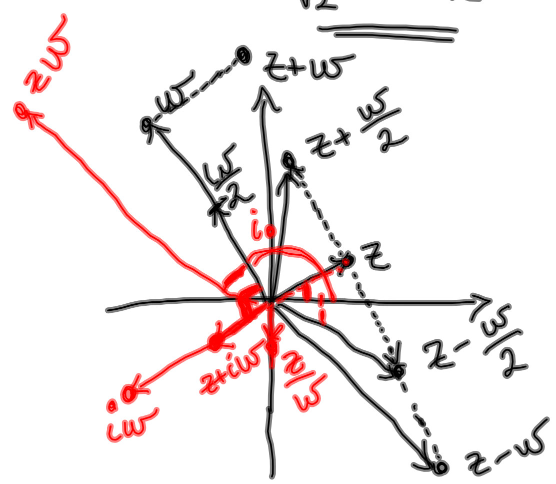


5.) b) $r=1,\ \theta=\frac{\pi}{4}$: $\quad z=a+ib$

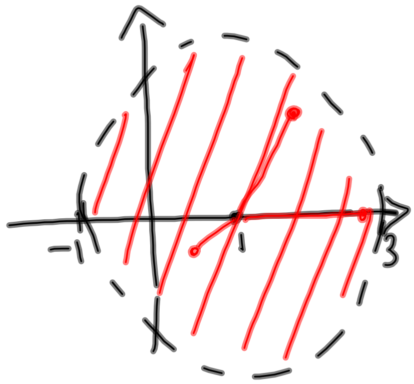
$$a=r\cos\theta=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$
$$b=r\sin\theta=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

$$\underline{\underline{z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}}}$$

9.)



10) b) $\{z : |z-1| < 2\}$



d) $\{z : |z-2| < |z-i+2|\}$

(Eks. 3.2.6)

$|z-2| < |z-i+2|$, la $\boxed{z = x + iy}$, $x \in \mathbb{R}$, $y \in \mathbb{R}$

$|z-2| = \sqrt{(x-2)^2 + y^2}$.

$|z-i+2| = \sqrt{(x+2)^2 + (y-1)^2}$.

$\sqrt{(x-2)^2 + y^2} < \sqrt{(x+2)^2 + (y-1)^2}$

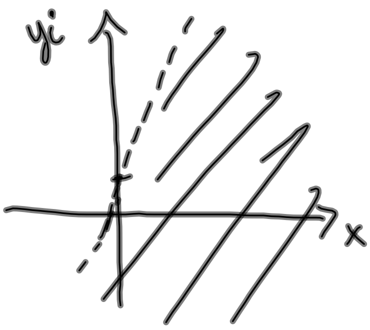
$\Updownarrow$ (siden $\sqrt{\dots}$ er ≥ 0)

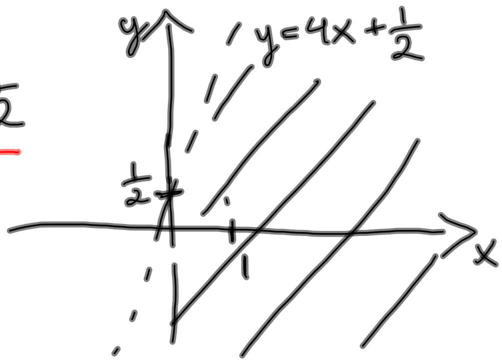
$(x-2)^2 + y^2 < (x+2)^2 + (y-1)^2$.

$\cancel{x^2} - 4x + \cancel{4} + \cancel{y^2} < \cancel{x^2} + 4x + \cancel{4} + \cancel{y^2} - 2y + 1$

$2y < 8x + 1$

$\underline{y < 4x + \frac{1}{2}}$





13.) $z = 1 + i\sqrt{3}$, $w = 1 + i$

a) $zw = (1 - \sqrt{3}) + i(1 + \sqrt{3})$

$$\frac{z}{w} = \frac{1+\sqrt{3}}{2} + i\,\frac{\sqrt{3}-1}{2}$$

b) z: $r_z = \sqrt{1+3} = 2$, $2\cos\theta_z = 1 \Rightarrow \cos\theta_z = \frac{1}{2}$

$2\sin\theta_z = \sqrt{3} \Rightarrow \sin\theta_z = \frac{\sqrt{3}}{2}$

$\Downarrow$

$\theta_z = \frac{\pi}{3}$

$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$w = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

c) $\frac{z}{w} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

Merk: $\frac{2}{\sqrt{2}} = \frac{(\sqrt{2})^{\cancel{2}}}{\cancel{\sqrt{2}}} = \sqrt{2}$

$r_{\frac{z}{w}} = \frac{r_z}{r_w}$, $\theta_{\frac{z}{w}} = \theta_z - \theta_w$

Fra a) vet vi at $\frac{z}{w} = \frac{\sqrt{3}+1}{2} + i\,\frac{\sqrt{3}-1}{2}$

$\frac{\sqrt{3}+1}{2} = \sqrt{2}\cos\frac{\pi}{12}$ og $\frac{\sqrt{3}-1}{2} = \sqrt{2}\sin\frac{\pi}{12}$

$\cos\frac{\pi}{12} = \underline{\frac{\sqrt{3}+1}{2\sqrt{2}}}$ og $\sin\frac{\pi}{12} = \underline{\frac{\sqrt{3}-1}{2\sqrt{2}}}$.

Gange m/ $\sqrt{2}$ oppe og nede:

$\cos\frac{\pi}{12} = \underline{\underline{\frac{\sqrt{6}+\sqrt{2}}{4}}}$, $\sin\frac{\pi}{12} = \underline{\underline{\frac{\sqrt{6}-\sqrt{2}}{4}}}$.