Plenum 08/11/13

Integrasjon : 8.6 : 1aef, 3, 5c, 7bce, 9, 11ac, 15, 26

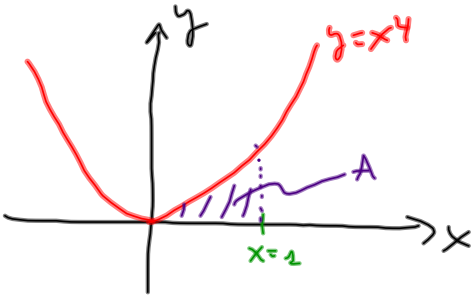
9.1 : 1abef, 5, 9, 11

9.2 : 1abcd

8.6

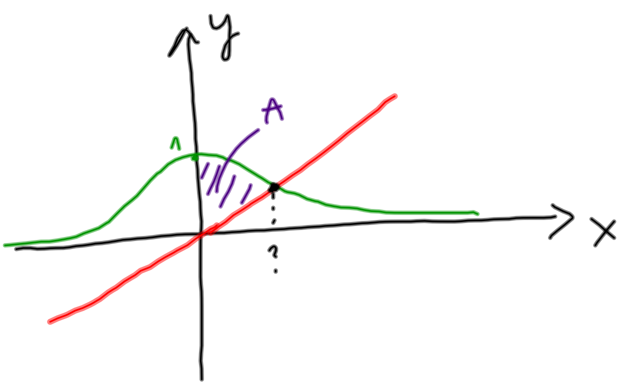
1) Finn arealet av området avgrenset av de oppgitte kurvene

a) $y = x^4$, x-aksen, $x=1$



$$A = \int_0^1 x^4 \, dx = \frac{1}{5} \cdot x^5 \Big|_{x=0}^{x=1} = \frac{1}{5}$$

f) $y = \frac{1}{1+x^2}$, $y = \frac{x}{2}$, y-aksen :



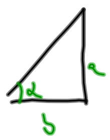
$\boxed{?}$ $\frac{1}{1+x^2} = \frac{x}{2} \iff 2 = x + x^3$

$x^3 + x \quad 2 = 0$

Vi ser at $x = 1$ er en rot.

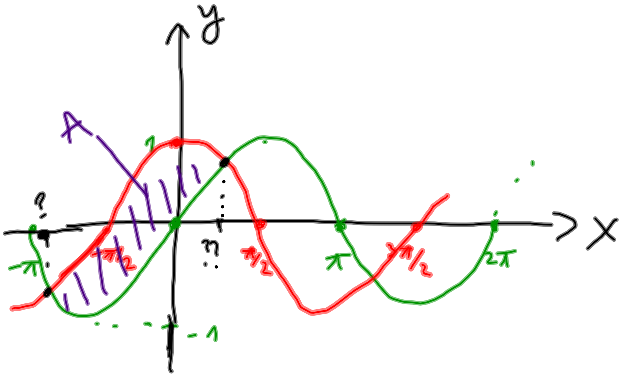
$$A = \int_0^1 \frac{1}{1+x^2}\,dx - \int_0^1 \frac{x}{2}\,dx = \arctan \Big|_{x=0}^{x=1} - \frac{1}{4}x^2 \Big|_{x=0}^{x=1} = \arctan 1 - \underbrace{\arctan 0}_{=0} - \frac{1}{4}$$

$$= \frac{\pi}{4} - \frac{1}{4} = \frac{\pi - 1}{4}$$



$\tan \alpha = \frac{a}{b} = 1$

3) Finne arealet mellom $\sin x$ og $\cos x$.



!?] $\sin x = \cos x \iff x = \frac{\pi}{4} + k\cdot\pi$

$(\tan x = 1)$ $\quad k \in \mathbb{Z}$

? $x = \frac{\pi}{4} - \pi = \frac{-3\pi}{4}$ $(k=-1)$

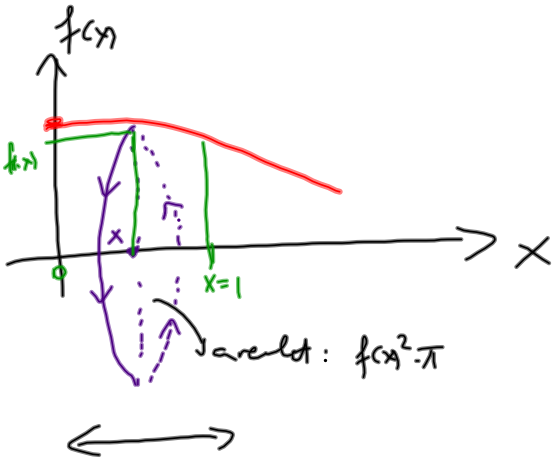
?? $x = \frac{\pi}{4}$ $(k=0)$

$$A = \int_{-3\pi/4}^{\pi/4} \cos x \, dx - \int_{-3\pi/4}^{\pi/4} \sin x \, dx = \sin x \Big|_{x=-3\pi/4}^{x=\pi/4} + \cos x \Big|_{x=-3\pi/4}^{x=\pi/4} = \sin \tfrac{\pi}{4} - \sin\tfrac{-3\pi}{4} + \cos\tfrac{\pi}{4} - \cos\tfrac{-3\pi}{4}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \underline{\underline{2\sqrt{2}}}$$

5) Finn volumet til omdreiningslegemet når vi dreier grafen om x-aksen.

$f(x) = \dfrac{1}{\sqrt{1+x^2}}$, $x=0$, $x=1$.

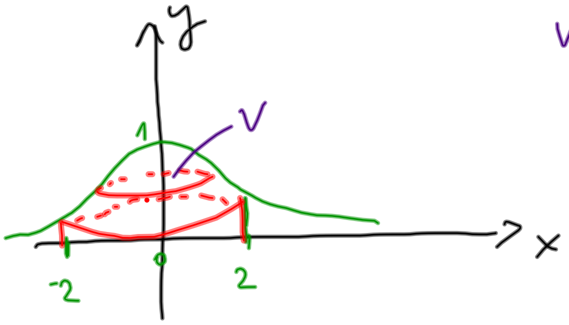


$$V = \int_0^1 \pi \cdot f(x)^2\,dx = \pi \int_0^1 \frac{1}{1+x^2}\,dx$$

$$= \pi \arctan \Big|_{x=0}^{x=1} = \pi \arctan 1 - \underbrace{\pi \arctan 0}_{=0}$$

$$= \pi \cdot \frac{\pi}{4} = \pi^2/4$$

7) c) $y = \frac{1}{1+x^2}$, $x=0$, $x=2$



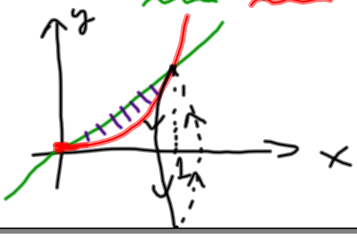
$$V = \int_0^2 2\pi \cdot x \cdot f(x)\,dy = 2\pi \int_0^2 x \cdot \frac{1}{1+x^2}\,dy$$

$$= 2\pi \int_0^2 \frac{x}{1+x^2}\,dx = \pi \int_0^2 \frac{2x}{1+x^2}\,dx = \circledast$$

den derivete er $2x$

$$\circledast = \pi \cdot \ln(1+x^2)\Big|_{x=0}^{x=2} = \pi \cdot \ln 5 - \pi \underbrace{\ln(1)}_{=0} = \underline{\underline{\pi \ln 5}}$$

9) a) $y = x$, $y = x^2$



$\int_a^b \pi f(x)^2\,dx$

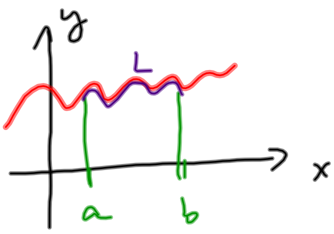
$$V = \int_0^1 \pi (x)^2\,dx - \int_0^1 \pi (x^2)^2\,dx = \pi \frac{x^3}{3}\Big|_0^1 - \pi \frac{x^5}{5}\Big|_0^1$$

$$= \pi \cdot \frac{1}{3} - \pi \cdot \frac{1}{5} = \frac{2\pi}{15}$$ ve.

11) Buelengde:

c) $y = \frac{x^2}{2} - \frac{1}{4}\ln x \quad , \quad x = 1, \quad x = e.$

$f'(x) = x - \frac{1}{4}\cdot\frac{1}{x}$



$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx = \int_1^e \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\, dx =$$

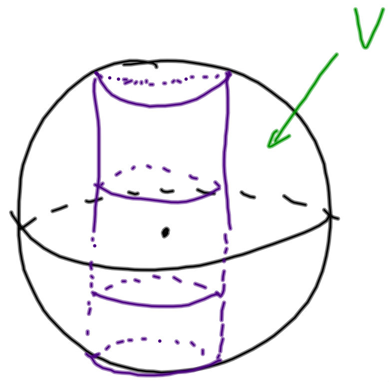
8.6.7

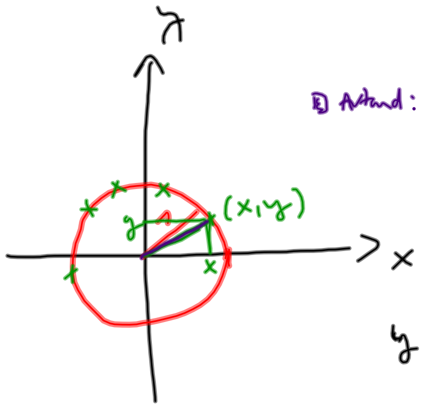
$$= \int_1^e \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\, dx = \int_1^e \sqrt{\frac{1}{2} + x^2 + \frac{1}{16x^2}}\, dx$$

$$= \int_1^e \sqrt{\frac{8x^2 + 16x^4 + 1}{16x^2}}\, dx = \int_1^e \sqrt{\frac{(4x^2+1)^2}{16x^2}}\, dx = \int_1^e \frac{4x^2+1}{4x}\, dx$$

$$= \int_1^e \left(x + \frac{1}{4x}\right) dx = \frac{x^2}{2}\Bigg|_1^e + \frac{1}{4}\cdot \ln x\Bigg|_1^e = \frac{e^2 - 1}{2} + \frac{1}{4}\cdot(\underbrace{\ln e}_{1} - \underbrace{\ln 1}_{0}) = \frac{e^2-1}{2} + \frac{1}{4}$$

15)

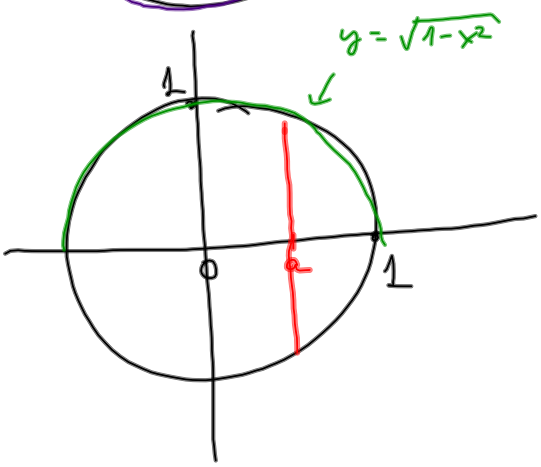




Avstand: $|(x,y)-(0,0)| =$

$= |(x,y)| = x^2+y^2 = 1$

$y = \pm\sqrt{1-x^2}$



$$V = 2 \cdot \int_a^1 2\pi\, x \cdot \overbrace{\underbrace{\sqrt{1-x^2}}_{\text{den deriverte} = -2x}}^{y}\, dx =$$

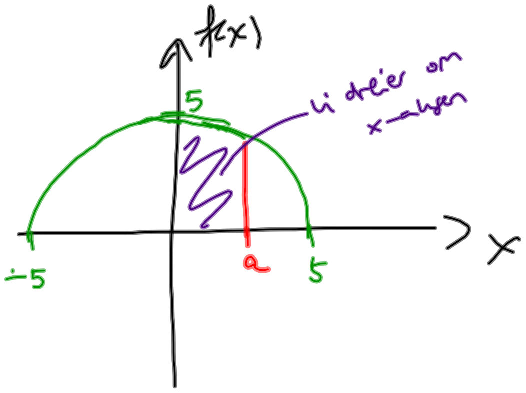
$$= -2\pi \int_a^1 -2x\,(1-x^2)^{1/2}\,dx =$$

$$= -2\pi\,\frac{(1-x^2)^{1/2+1}}{\frac{1}{2}+1}\Bigg|_{x=a}^{x=1} = \frac{-4\pi}{3}\,(1-x^2)^{3/2}\Bigg|_{x=a}^{x=1}$$

$$= \frac{4\pi}{3}\cdot(1-a^2)^{3/2}$$

26) La $a \in (0,5)$. Området avgrenset av x-aksen, y-aksen, grafen til $f(x)=\sqrt{25-x^2}$ og linjen $x=a$ dreies om x-aksen.

a) Volumet til omdreiningslegemet, uttrykt ved a.

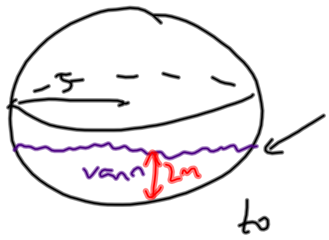


$$V=\int_0^a \pi \cdot f(x)^2\,dx = \int_0^a \pi\cdot(25-x^2)\,dx$$

$$= 25\pi\, x\Big|_0^a - \pi \frac{x^3}{3}\Big|_0^a =$$

$$= \pi a\left(25-\frac{1}{3}a^2\right) \text{ ke.}$$

b)



akk. nå $= t_0$ tømmes vann på $0.5\ m^3/min$.

– Hvor fort avtar vanndybden?

Vi ser på a som funksjon av t

$a(t)$ $\quad a(t_0)=3$

$$V(t) = \pi\left(25\,a(t) - \frac{a(t)^3}{3}\right)$$

$$V'(t) = 25\pi\, a'(t) - a(t)^2\cdot a'(t)\,,\ \text{tid der } a(t)=3 \text{ er } V'(t)=\tfrac{1}{2}.$$

$$\frac{1}{2} = \pi\, a'(t) \cdot (25-9)$$

$$a'(t) = \frac{1}{2\pi \cdot 16}$$

Så vannhøyden avtar med $\boxed{\frac{1}{32\pi}\ \text{m/min}}$ på dette tidspunktet.

nov 8-09:37

Delvis integrasjon

11) $$\int \frac{x^2 \arctan x}{1+x^2}\,dx = uv - \int v \cdot u' = (x - \arctan x)\arctan x - \underbrace{\int \frac{x - \arctan x}{1+x^2}\,dx}_{= I}$$

2

$u = \arctan x \qquad u' = \frac{1}{1+x^2}$

$v' = \frac{x^2}{1+x^2} \qquad v = x - \arctan x$

1

Først, finner vi v:

$$v = \int \frac{x^2}{1+x^2}\,dx = \int dx - \int \frac{dx}{1+x^2} = x - \arctan x.$$

$$\begin{array}{l|l} x^2 & x^2+1 \\ \hline -\;x^2+1 & 1 \\ \hline 0 - 1 & \end{array} \Rightarrow \frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$I = \int \frac{x - \arctan x}{1+x^2}\,dx = \int \frac{x}{1+x^2}\,dx - \int \frac{\arctan x}{1+x^2}\,dx$$

$$= \frac{1}{2}\cdot \int \frac{2x}{1+x^2}\,dx - \int \arctan x \cdot \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{2}\cdot \ln(1+x^2) - \frac{(\arctan x)^2}{2} + C.$$

til sammen:

$$\int \frac{x^2 \arctan x}{1+x^2}\,dx = \arctan x\,(x - \arctan x) - \frac{1}{2}\ln(1+x^2) + \frac{(\arctan x)^2}{2} + C$$

$$= x\cdot \arctan x - \frac{1}{2}\arctan^2 x - \frac{1}{2}\ln(1+x^2) + C$$

9.2. Substitusjon

2) b) $\displaystyle\int \frac{\sqrt{x}}{1+x}\,dx = \int \frac{u}{1+u^2}\cdot 2u\,du = \int \frac{2u^2}{1+u^2}\,du =$

$u = \sqrt{x} \rightsquigarrow x = u^2$

$dx = 2u\,du$

$= 2u - 2\arctan u + C$

$= 2\sqrt{x} - 2\arctan\sqrt{x} + C$

shift tilbake!

nov 8-09:50

9) $\int \cos(\ln x)\,dx = \int \cos(u)\, e^u\, du = (*)$

$u = \ln x \rightsquigarrow x = e^u$

$dx = e^u\,du.$

$w = \cos(u) \quad w' = -\sin(u)$

$v' = e^u \quad v = e^u$

$(*) = \cos u \cdot e^u + \int \sin u \cdot e^u\, du = \cos u\, e^u + \sin u\, e^u - \int \cos u \cdot e^u\, du$

$w = \sin u \quad w' = \cos u$

$v' = e^u \quad v = e^u$

$\int \cos u\, e^u\, du = \frac{1}{2}\left(\cos u\, e^u + \sin u\, e^u\right) + C = \underline{\frac{1}{2} x \left(\cos(\ln x) + \sin(\ln x)\right) + C}$

skift tilbake