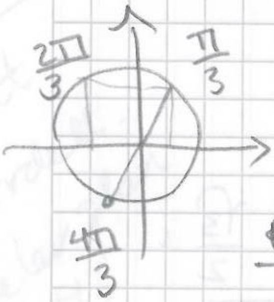
3.5:

13) $z = -1 + i\sqrt{3}$:

Polarform: $|z| = \sqrt{1+3} = \underline{2}$

θ: $-1 = 2\cos\theta$ $\Rightarrow$ $\cos\theta = -\frac{1}{2}$ $\Rightarrow$ $\theta =$

$\sqrt{3} = 2\sin\theta$ $\quad \sin\theta = \frac{\sqrt{3}}{2}$



$$z = \underline{2e^{\frac{2\pi}{3}i}}$$

Rot 1:

$$w_0 = z^{\frac{1}{2}} = \underline{\underline{\sqrt{2}\,e^{\frac{\pi}{3}i}}}$$

$$= \sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

Må gange med $e^{i\frac{2\pi}{2}} = e^{i\pi}$ for neste rot:

$$w_1 = \sqrt{2}\,e^{\frac{\pi}{3}i}e^{\pi i} = \underline{\underline{\sqrt{2}\,e^{\frac{4\pi}{3}i}}}$$

$$= \underline{\underline{\sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)}}$$

b) $z^4 + z^2 + 1 = 0$

$w := z^2 \Rightarrow w^2 + w + 1 = 0$

$$w = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2}$$

$$= \begin{cases} -\frac{1}{2} + \frac{\sqrt{3}}{2}i \\ -\frac{1}{2} - \frac{\sqrt{3}}{2}i \end{cases}$$

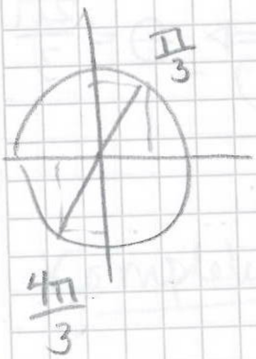
Så

$z^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ eller $z^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\overline{w_0} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$, polarform: $|\overline{w_0}| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

θ: $-\frac{1}{2} = \cos\theta$

$-\frac{\sqrt{3}}{2} = \sin\theta$ $\Rightarrow$ $\theta = \frac{4\pi}{3}$



$$\overline{w_0} = e^{\frac{4\pi}{3}i}$$

Rot 1: $z = \underline{\underline{e^{\frac{2\pi}{3}i}}} = w_1 = \underline{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}$

Finner neste rot:

Rot 2:

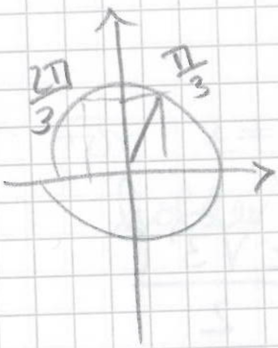
$$z = e^{\frac{2\pi}{3}i} e^{\frac{2\pi}{2}i} = e^{i\left(\frac{2\pi}{3} + \pi\right)}$$

$$= \underline{\underline{e^{\frac{5\pi}{3}i}}} = w_3$$

$\overline{w_1} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, polarform: $|\overline{w_1}| = |\overline{w_0}| = 1$

θ: $-\frac{1}{2} = \cos\theta$

$\frac{\sqrt{3}}{2} = \sin\theta$ $\Rightarrow$ $\theta = \frac{2\pi}{3}$



$$\underline{\overline{w_1} = e^{\frac{2\pi}{3}i}}$$

Rot 1: $z = \underline{\underline{e^{\frac{\pi}{3}i}}} = w_0 = \underline{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$

Rot 2: $z = e^{\left(\frac{\pi}{3} + \pi\right)i} = \underline{\underline{e^{\frac{4\pi}{3}i}}} = w_2$