3.5.3) c) $z^6 - 4z^3 + 4 = w^2 - 4w + 4 = 0$

$$\boxed{w = z^3}$$

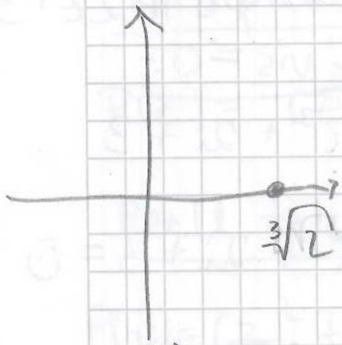
$$w = \frac{4 \pm \sqrt{16 - 4 \cdot 4}}{2} = \frac{4 \pm 0}{2} = \underline{2}$$

$$\Rightarrow \quad z^3 = 2 = 2e^{i2\pi} \quad \Rightarrow \quad w_0 = \sqrt[3]{2} \text{ er Rot 1}$$

Rot 1:

$$w_1 = (2e^{i2\pi})^{\frac{1}{3}}$$

$$= \underline{\sqrt[3]{2}\, e^{\frac{2\pi}{3} i}} = \sqrt[3]{2}\left(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right) = \sqrt[3]{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$$

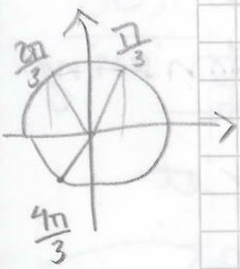


Må gange m/ $e^{\frac{2\pi}{3} i}$ for neste rot:

Rot 2:

$$w_2 = w_1 e^{\frac{2\pi}{3} i} = \sqrt[3]{2}\, e^{\left(\frac{2\pi + 2\pi}{3}\right) i}$$

$$= \underline{\sqrt[3]{2}\, e^{\frac{4\pi}{3} i}} = \sqrt[3]{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$



evt. bare reelt polynom $\Rightarrow$ kompleks konjugert er også rot

Har 3 røtter $w_0 = \sqrt[3]{2}$, $w_1 = \sqrt[3]{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$ og $w_2 = \sqrt[3]{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$.

Siden alle røttene har multiplisitet 1 i $w^2 - 4w + 4$, må alle ha multiplisitet 2 i $z^6 - 4z^3 + 4$.

Kompleks faktorisering:

$$z^6 - 4z^3 + 4 = (z - \sqrt[3]{2})^2 \left(z - \sqrt[3]{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\right)^2 \cdot \left(z - \sqrt[3]{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)\right)^2$$

Reell faktorisering: $z^6 - 4z^3 + 4 =$

$$= (z - \sqrt[3]{2})^2 (z^2 + \sqrt[3]{2}\, z + 2^{\frac{2}{3}})^2$$

$$|w_1| = \left|\sqrt[3]{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\right|$$
$$= \sqrt[3]{2} \sqrt{\frac{1}{4} + \frac{3}{4}}$$
$$= \sqrt[3]{2}$$

$$(z - w)(z - \overline{w})$$
$$= z^2 - \overline{w} z - wz + w\overline{w}$$
$$= z^2 - z(w + \overline{w}) + |w|^2$$
$$= z^2 - 2\mathrm{Re}(w) z + |w|^2$$
$$\in \mathbb{R}$$

5.) a) Vis: i er rot i

$$P(z) = z^4 + 2z^3 + 4z^2 + 2z + 3$$

Sett inn i for z og vis $= 0$:

$$P(i) = i^4 + 2i^3 + 4i^2 + 2i + 3$$
$$= 1 - 2i - 4 + 2i + 3 = 0$$

Så i er en rot i $P(z)$.

b) Komplekse & reelle faktorisering:

i er en rot i $P(z)$ og $P(z)$ er et reelt polynom. Lemma 3.5.3 gir at $-i$ også er en rot i $P(z)$.

Dermed må $P(z)$ være delelig med $(z - i)(z + i)$

3. kvad. set. $= z^2 + 1$.

Gjør polynomdivisjonen:

$$z^4 + 2z^3 + 4z^2 + 2z + 3 \div z^2 + 1 = z^2 + 2z + 3$$
$$-(z^4 + z^2)$$
$$2z^3 + 3z^2 + 2z + 3$$
$$-(2z^3 + 2z)$$
$$3z^2 + 3$$
$$-(3z^2 + 3)$$
$$0$$