

4.3:

$$3.) a) \lim_{n \rightarrow \infty} (\sqrt{n+2} - \sqrt{n})$$

$$= \lim_{n \rightarrow \infty} \frac{(\sqrt{n+2} - \sqrt{n})(\sqrt{n+2} + \sqrt{n})}{\sqrt{n+2} + \sqrt{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{n+2-n}{\sqrt{n+2} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{2}{\sqrt{n+2} + \sqrt{n}}$$

$$= \underline{\underline{0}}$$

$$b) \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+\sqrt{n}} - \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\sqrt{n}} + \sqrt{n}}{(\sqrt{n+\sqrt{n}} + \sqrt{n})(\sqrt{n+\sqrt{n}} - \sqrt{n})}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{n+\sqrt{n}} + \sqrt{n}}{n+\sqrt{n}-n} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+\sqrt{n}} + \sqrt{n}}{\sqrt{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n+\sqrt{n}} + \sqrt{n}}{\sqrt{n}}}{\frac{\sqrt{n}}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{\frac{n+\sqrt{n}}{n}} + 1}{1}$$

$$= \lim_{n \rightarrow \infty} \left(\sqrt{1 + \frac{1}{\sqrt{n}}} + 1 \right) = 1 + 1 = \underline{\underline{2}}$$

