altså et lukket og begrenset intervall.

5.4: Grenseverdier

Bruker regneregler for grenseverdier

$\boxed{1)}$ a) $\lim\limits_{x \to 0} \dfrac{3x^2+7}{3x+\cos x} = \underline{\underline{7}}$

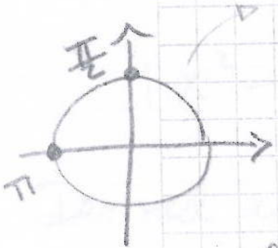
$\cos x \xrightarrow[x \to 0]{} 1$

b) $\lim\limits_{x \to 0^+} \dfrac{x^4+\sqrt{x}+e^{x^2}}{7+\sin(\sqrt{x})} = \underline{\underline{\dfrac{1}{7}}}$

$\tan = \dfrac{\sin}{\cos}$, $\sin\pi = 0$, $\cos\pi = -1$

$\sin(\frac{\pi}{2}) = 0$

$\boxed{c)}$ $\lim\limits_{x \to \pi} \dfrac{\tan x + 3\cos x}{\sin(\frac{x}{2})+4} = \underline{\underline{-\dfrac{3}{5}}}$



$\boxed{2)}$ $\boxed{a)}$ Vis: $\lim\limits_{x \to 2} 3x = 6$ fra definisjonen.

La $\varepsilon > 0$ være gitt.

Merk at: $|3x-6| = 3|x-2| = 2|h|$

Definer $h := x-2$. Se at hvis $|h| < \frac{\varepsilon}{2}$, så er $|3x-6| = 2|h| < \varepsilon$.

Velg derfor $\delta = \frac{\varepsilon}{2}$. Da vil, for alle x s.a. $|x-2| = |h| < \delta$ det være slik at:

$$|3x-6| = 2|h| < 2\delta = 2\frac{\varepsilon}{2} = \varepsilon,$$

$= |f(x) - b|$ i Def. 5.4.1

så dermed er $\lim\limits_{x \to 2} 3x = 6$ fra Def. 5.4.1.

$\lim_{x \to 3} x^2 = 9$ fra definisjonen:

b) La $\varepsilon > 0$ være gitt.
La $h = x - 3$.
Merk at $(|f(x) - b| =)\ |x^2 - 9| = |(x-3)(x+3)|$

$$= |h||x+3| = |h||h+6|$$

Hvis $|h| < 1$, så vil $|h+6| < 7$. Så ved å velge
$|h| < \min\{\frac{\varepsilon}{7}, 1\}$ er: $|x^2 - 9| = |h||h+6| < \delta \cdot 7 \leq \frac{\varepsilon}{7} 7 = \varepsilon$

Velg $\delta = \min\{\frac{\varepsilon}{7}, 1\}$. Da vil, for alle x s.a.
$|x - 3| = |h| < \delta$, det være s.a.

$$|x^2 - 9| = |h||h+6| < \varepsilon$$

Dermed er $\lim_{x \to 3} x^2 = 9$ fra definisjonen.

3.) a) $\lim_{x \to 0} \frac{7x^2 + 4x^4}{3x^3 - 2x^2} = \lim_{x \to 0} \frac{\cancel{x^2}(7 + 4x^2)}{\cancel{x^2}(3x - 2)} = \underline{\underline{-\frac{7}{2}}}$

b) $\lim_{x \to \infty} \frac{8x^2 + 2x + 7}{\sqrt{x} - 4x^2}$

$$= \lim_{x \to \infty} \frac{8 + \frac{2}{x} + \frac{7}{x^2}}{\frac{1}{x^{\frac{3}{2}}} - 4} = -\frac{8}{4} = \underline{\underline{-2}}$$

c) $\lim_{x \to \infty} (\sqrt{x^2 + 3x} - x) = \lim_{x \to \infty} \frac{(\sqrt{x^2 + 3x} - x)(\sqrt{x^2 + 3x} + x)}{\sqrt{x^2 + 3x} + x}$

$$= \lim_{x \to 0} \frac{x^2 + 3x - x^2}{\sqrt{x^2 + 3x} + x} = \lim_{x \to \infty} \frac{3}{\frac{\sqrt{x^2 + 3x}}{x} + 1}$$

$$= \lim_{x \to \infty} \frac{3}{\sqrt{\frac{x^2 + 3x}{x^2}} + 1} = \lim_{x \to \infty} \frac{3}{\sqrt{1 + \frac{3}{x}} + 1} = \underline{\underline{\frac{3}{2}}}$$