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c) $\lim_{x \to 0^+} \frac{\sin x}{x}$

Fra b):

$$\cos x < \frac{\sin x}{x} < 1 \quad , \quad x \in (0, \frac{\pi}{2})$$

$$\Downarrow$$

pga går til grense

$$\lim_{x \to 0^+} \cos x \leq \lim_{x \to 0^+} \frac{\sin x}{x} \leq \underbrace{\lim_{x \to 0^+} 1}_{=1}$$

Men $\lim_{x \to 0^+} \cos x = 1 \Rightarrow$

$$1 \leq \lim_{x \to 0^+} \frac{\sin x}{x} \leq 1$$

$\Downarrow$ ($\leq$ må holde m/ $=$)

$$\underline{\underline{\lim_{x \to 0^+} \frac{\sin x}{x} = 1}}$$

d) Kan gjøre tilsv. arg. & tegning når $x \in (-\frac{\pi}{2}, 0)$ for å vise $\lim_{x \to 0^-} \frac{\sin x}{x} = 1$.

Dermed, siden $\lim_{x \to 0^+} \frac{\sin x}{x} = \lim_{x \to 0^-} \frac{\sin x}{x} = 1$

eksisterer $\underline{\underline{\lim_{x \to 0} \frac{\sin x}{x} = 1}}$.

e) (i) $\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{3 \sin 3x}{3x}$

$$= 3 \lim_{x \to 0} \frac{\sin 3x}{3x} = 3 \lim_{y \to 0} \frac{\sin y}{y} = 3 \cdot 1 = \underline{\underline{3}}$$

$\boxed{y := 3x}$

$x \to 0 \Rightarrow$
$y = 3x \to 0$

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ii) $\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x \cos x}$

$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\substack{\downarrow \\ 1 \\ \text{når } x\to 0}} \cdot \underbrace{\frac{1}{\cos x}}_{\substack{\downarrow \\ 1 \\ \text{når } x\to 0}} = 1 \cdot 1 = \underline{\underline{1}}$

iii) $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)}$

$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2} \frac{1}{1+\cos x}$

$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\substack{\downarrow \\ 1}} \cdot \underbrace{\frac{\sin x}{x}}_{\substack{\downarrow \\ 1}} \cdot \underbrace{\frac{1}{1+\cos x}}_{\substack{\downarrow \\ \frac{1}{1+1} = \frac{1}{2}}} = 1 \cdot 1 \cdot \frac{1}{2} = \underline{\underline{\frac{1}{2}}}$

6.1 Derivasjon

Deriver:

1) e) $f(x) = \cos(e^x)$

$f'(x) = \underline{\underline{\sin(e^x)\, e^x}}$

3.) c) Logaritmisk derivasjon:

$f(x) = x^x$

$f'(x) = f(x) \cdot D[\ln|f(x)|]$

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$= x^x \cdot D[\ln|x^x|]$

Sett opp på noen slik overhold ... selv

$= x^x \cdot D[\ln(|x|^x)]$

$= x^x \cdot D[x \ln|x|]$